

Exercice 11 :

Soient les suites (u_n) définies par

$$u_{n+1} = u_n \times q \quad \text{et} \quad u_0 < 0$$

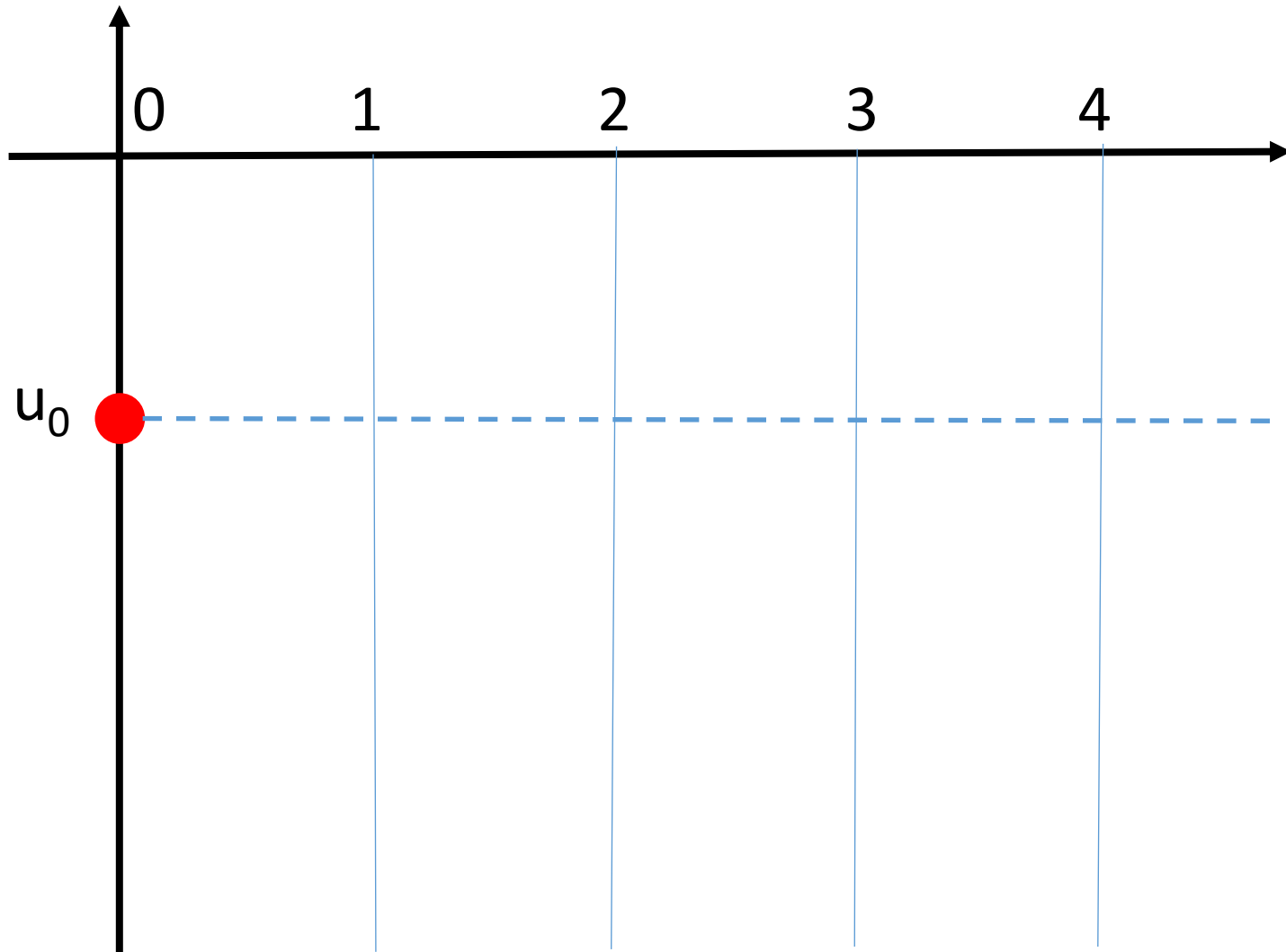
1°) Tracez (les 5 premiers points et sur deux repères différents pour $q \geq 0$ et $q < 0$) les courbes de toutes les suites, pour le même u_0 fixé, et pour tous les réels q possibles.

2°) Déduisez-en leurs sens de variations
et leurs limites.

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



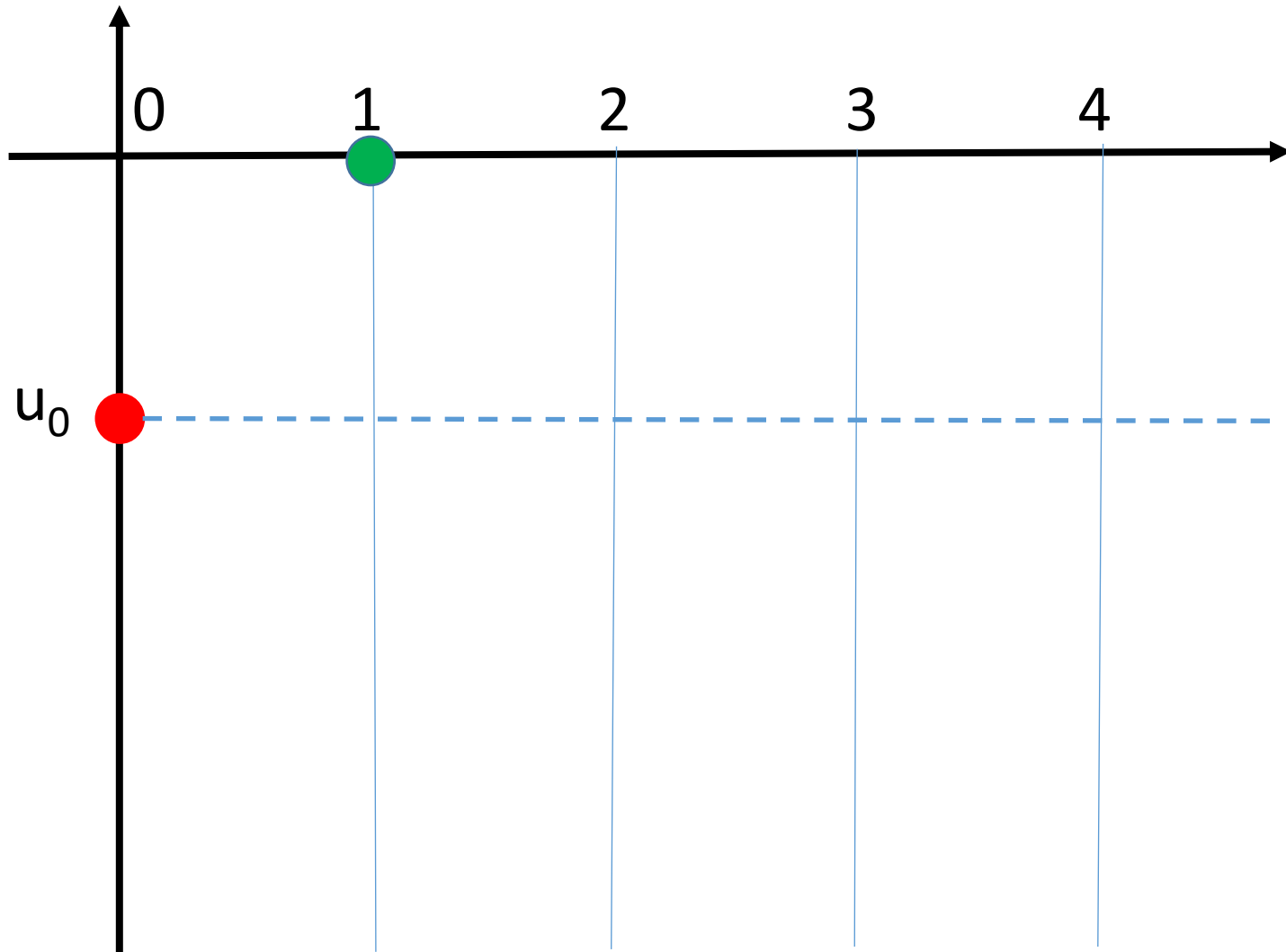
$$q = 0$$

$$u_0 < 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



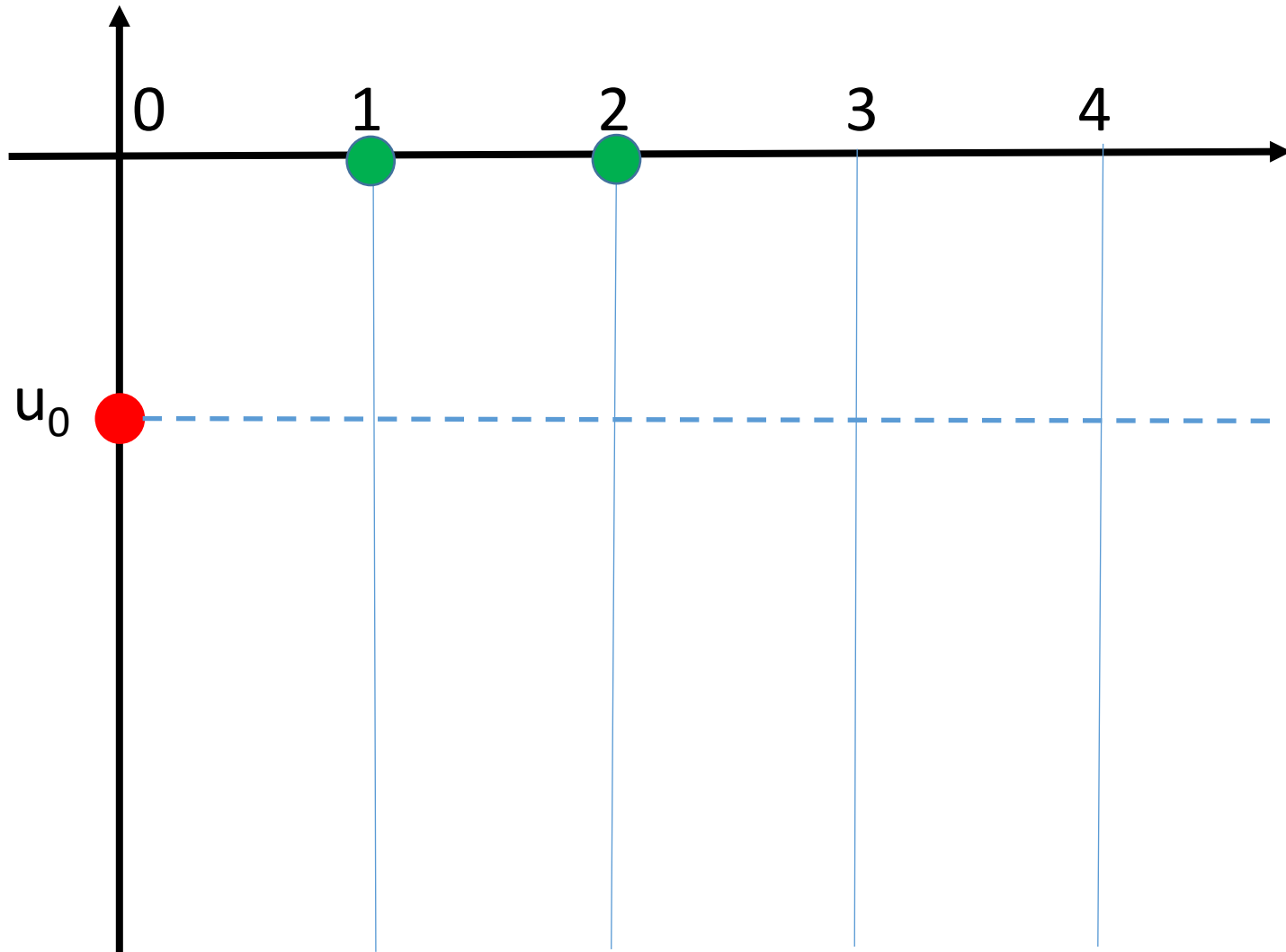
$$q = 0$$

$$u_1 = u_0 \times q = u_0 \times 0 = 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

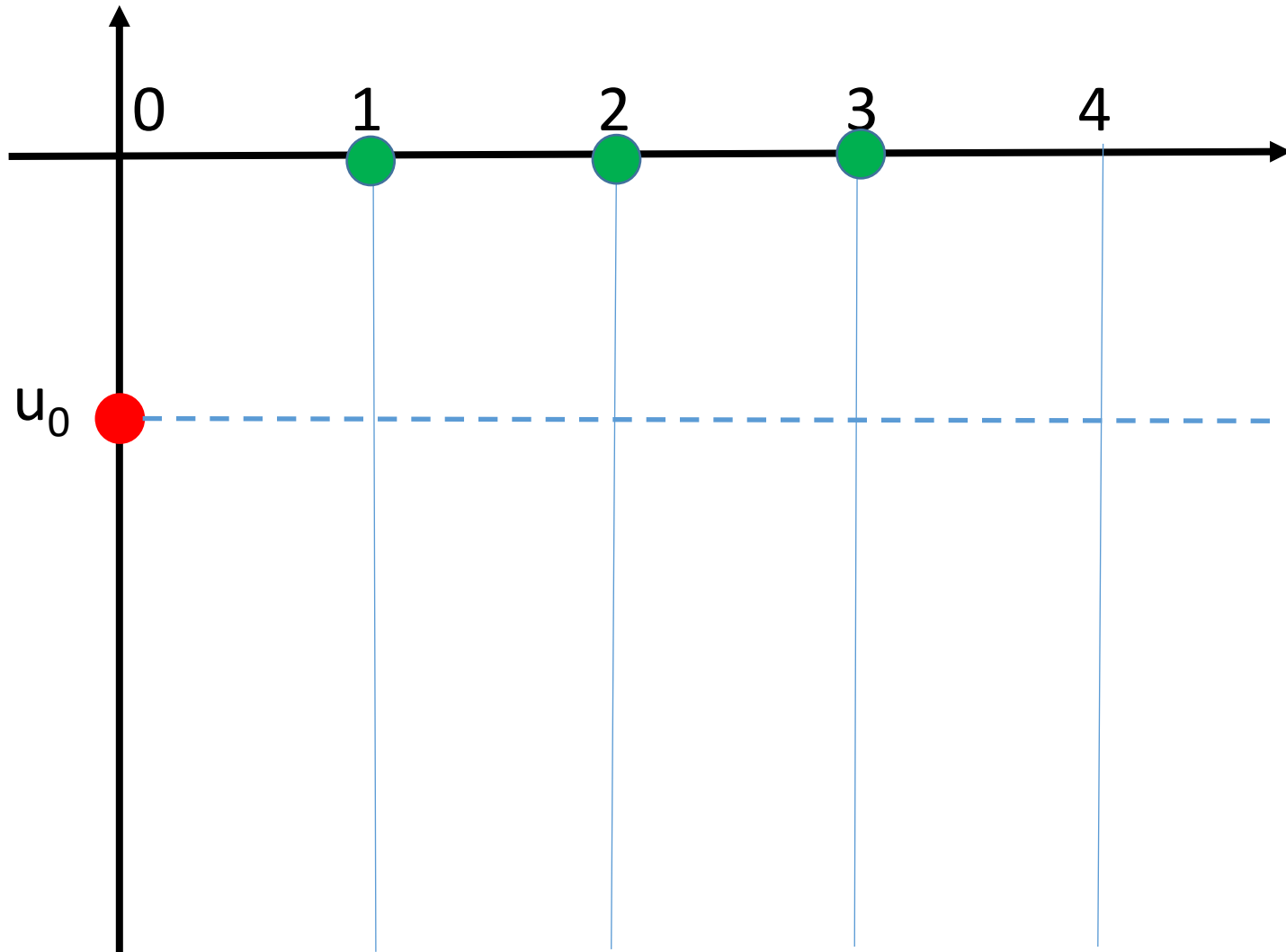
$$u_1 = u_0 \times q = u_0 \times 0 = 0$$

$$u_2 = u_1 \times q = 0 \times 0 = 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$u_1 = u_0 \times q = u_0 \times 0 = 0$$

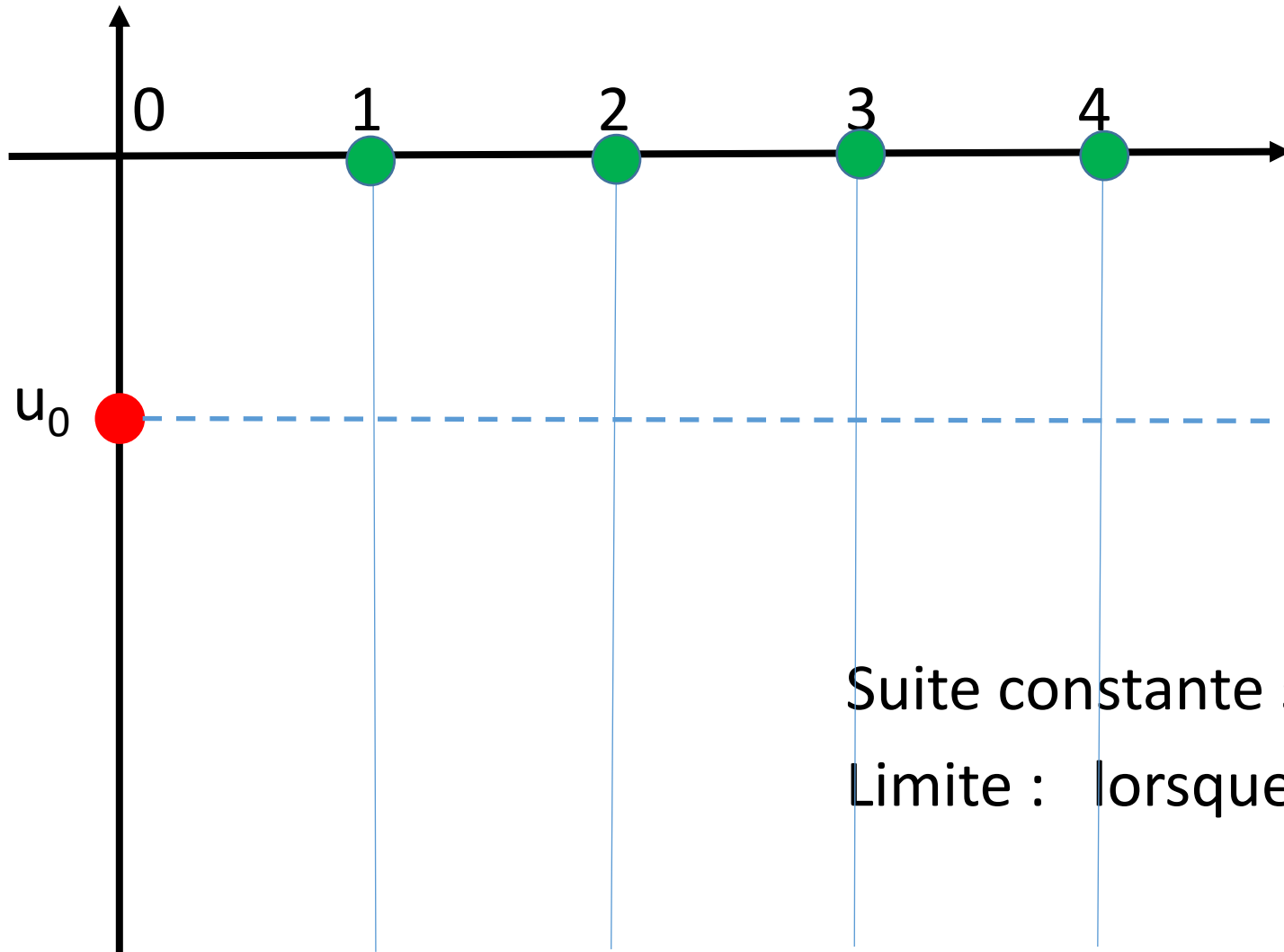
$$u_2 = u_1 \times q = 0 \times 0 = 0$$

$$u_3 = u_2 \times q = 0 \times 0 = 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$u_1 = u_0 \times q = u_0 \times 0 = 0$$

$$u_2 = u_1 \times q = 0 \times 0 = 0$$

$$u_3 = u_2 \times q = 0 \times 0 = 0$$

$$u_4 = u_3 \times q = 0 \times 0 = 0$$

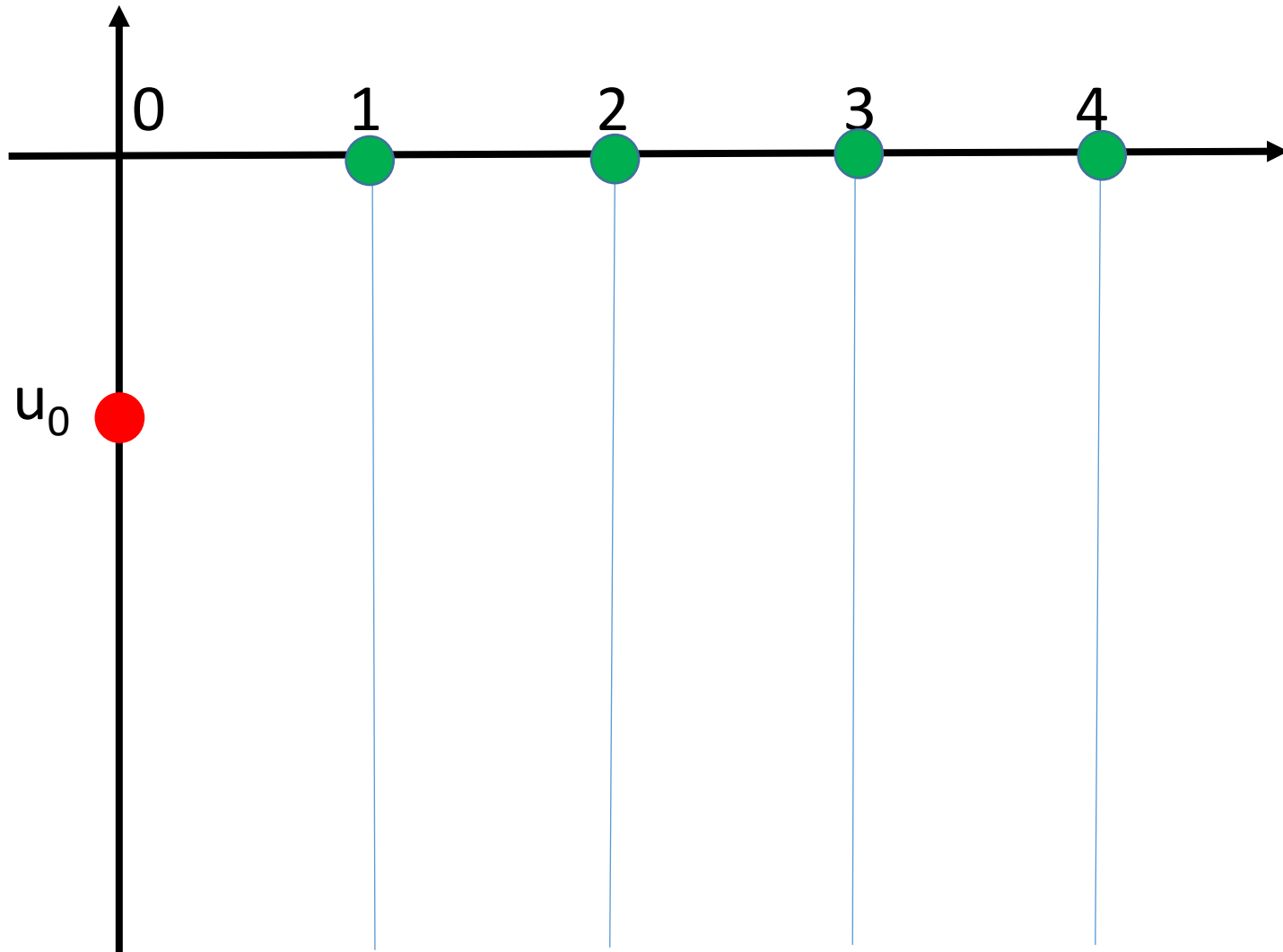
Suite constante sur $\mathbb{N}^*$

Limite : lorsque $n \rightarrow +\infty$ $u_n \rightarrow 0$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

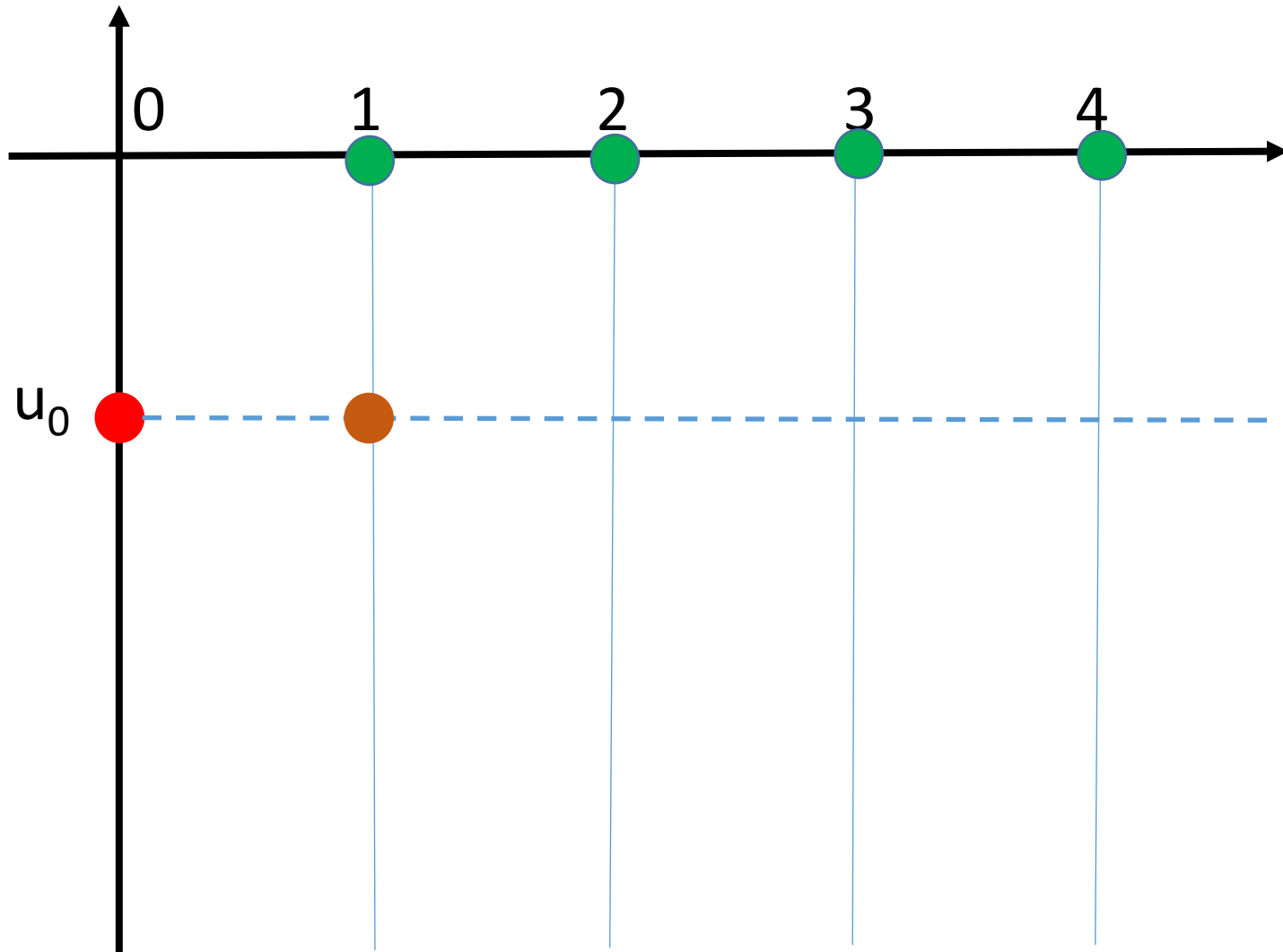
2^{ème} cas : $q = 1$

$$u_0 < 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

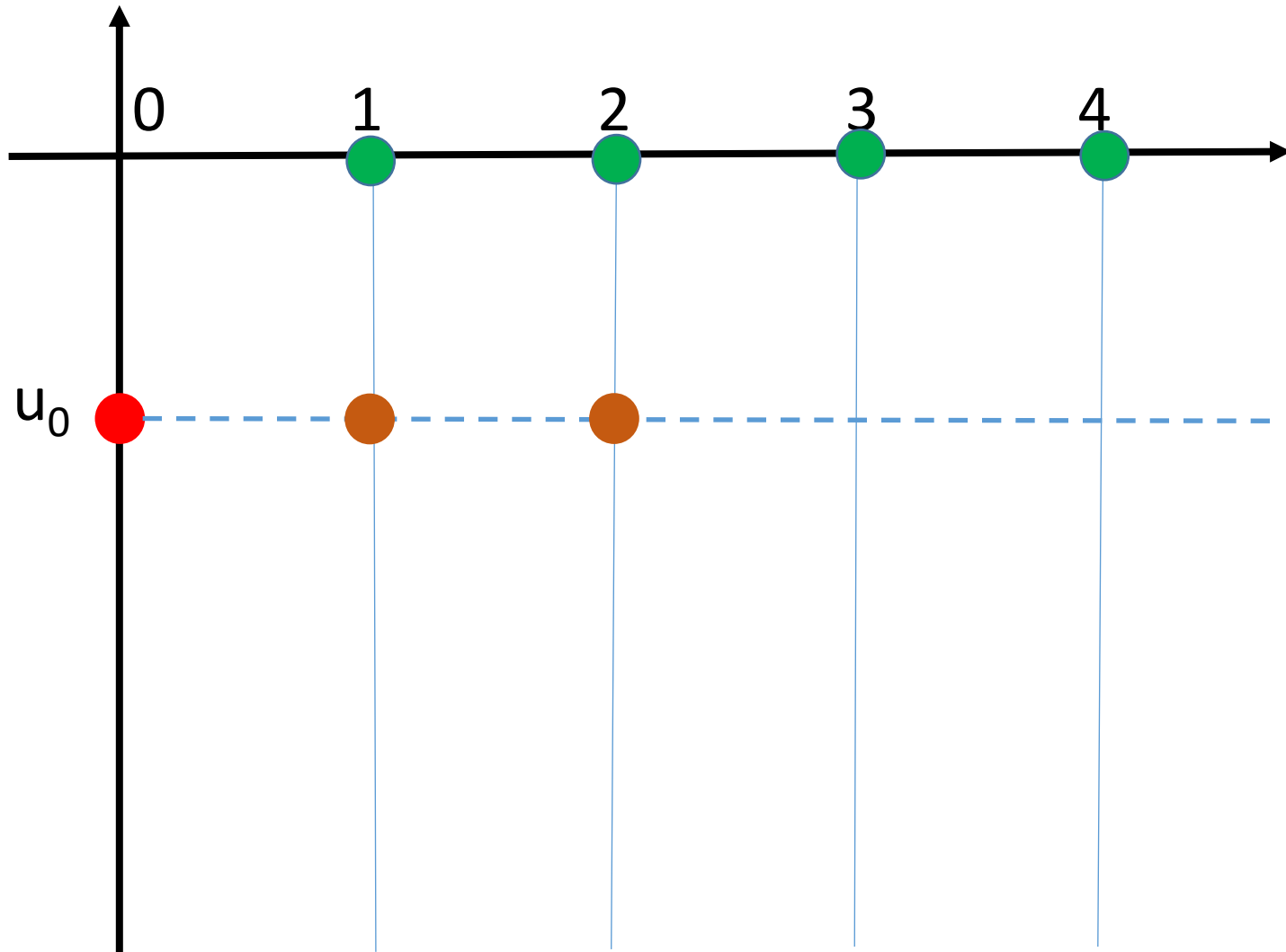
$$q = 1$$

$$u_1 = u_0 \times q = u_0 \times 1 = u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$q = 1$$

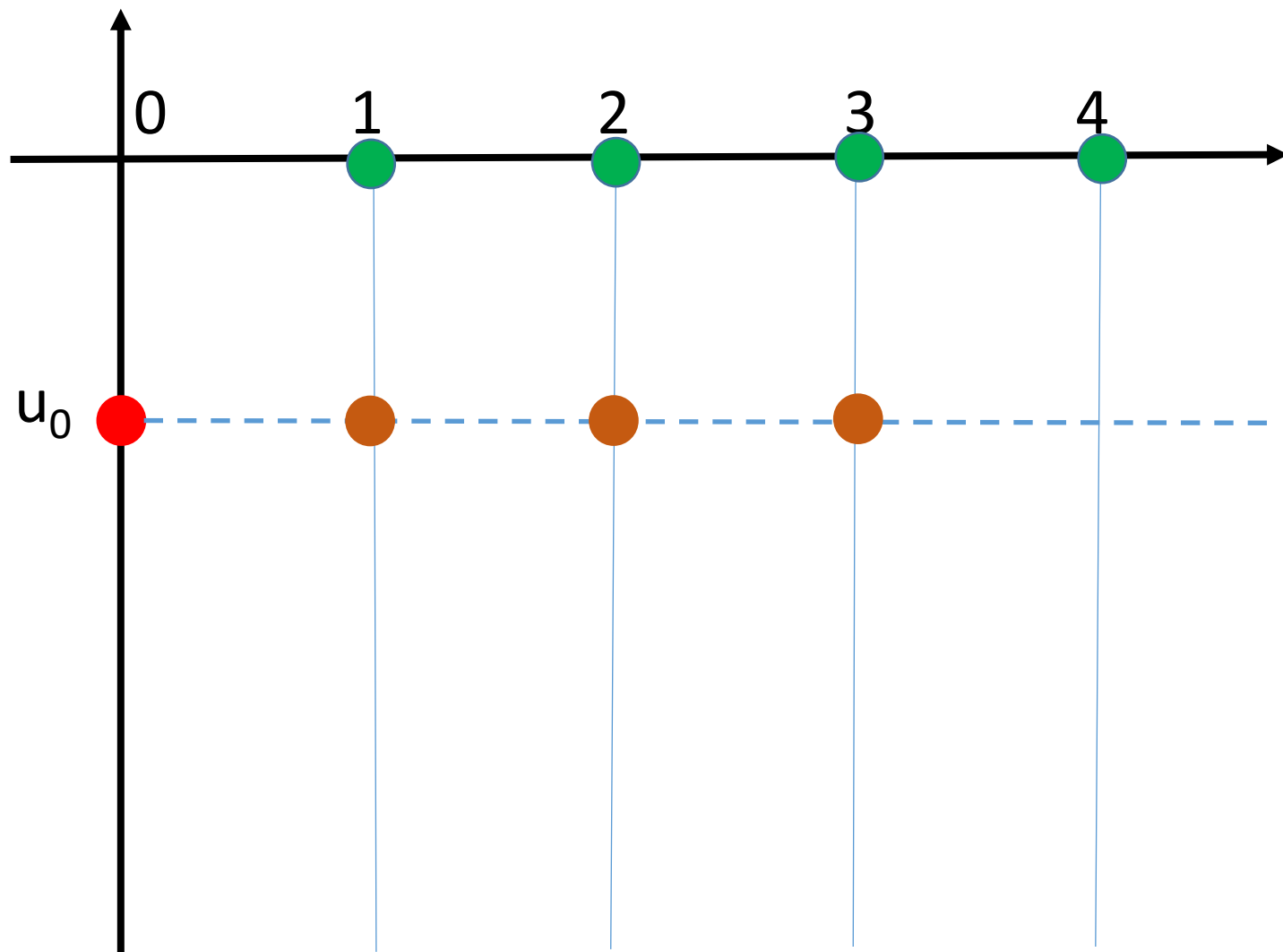
$$u_1 = u_0 \times q = u_0 \times 1 = u_0$$

$$u_2 = u_1 \times q = u_0 \times 1 = u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$q = 1$$

$$u_1 = u_0 \times q = u_0 \times 1 = u_0$$

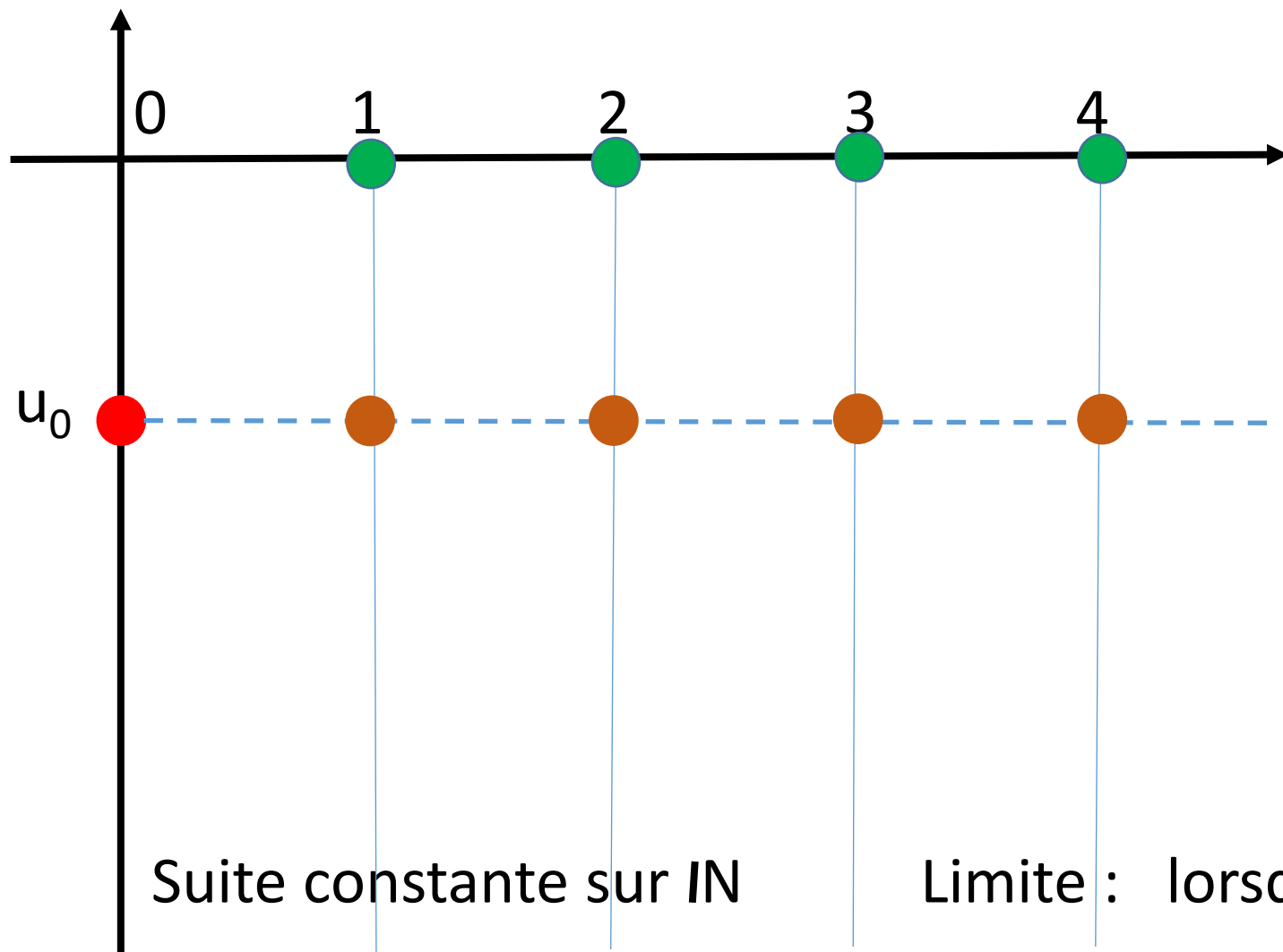
$$u_2 = u_1 \times q = u_0 \times 1 = u_0$$

$$u_3 = u_2 \times q = u_0 \times 1 = u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$q = 1$$

$$u_1 = u_0 \times q = u_0 \times 1 = u_0$$

$$u_2 = u_1 \times q = u_0 \times 1 = u_0$$

$$u_3 = u_2 \times q = u_0 \times 1 = u_0$$

$$u_4 = u_3 \times q = u_0 \times 1 = u_0$$

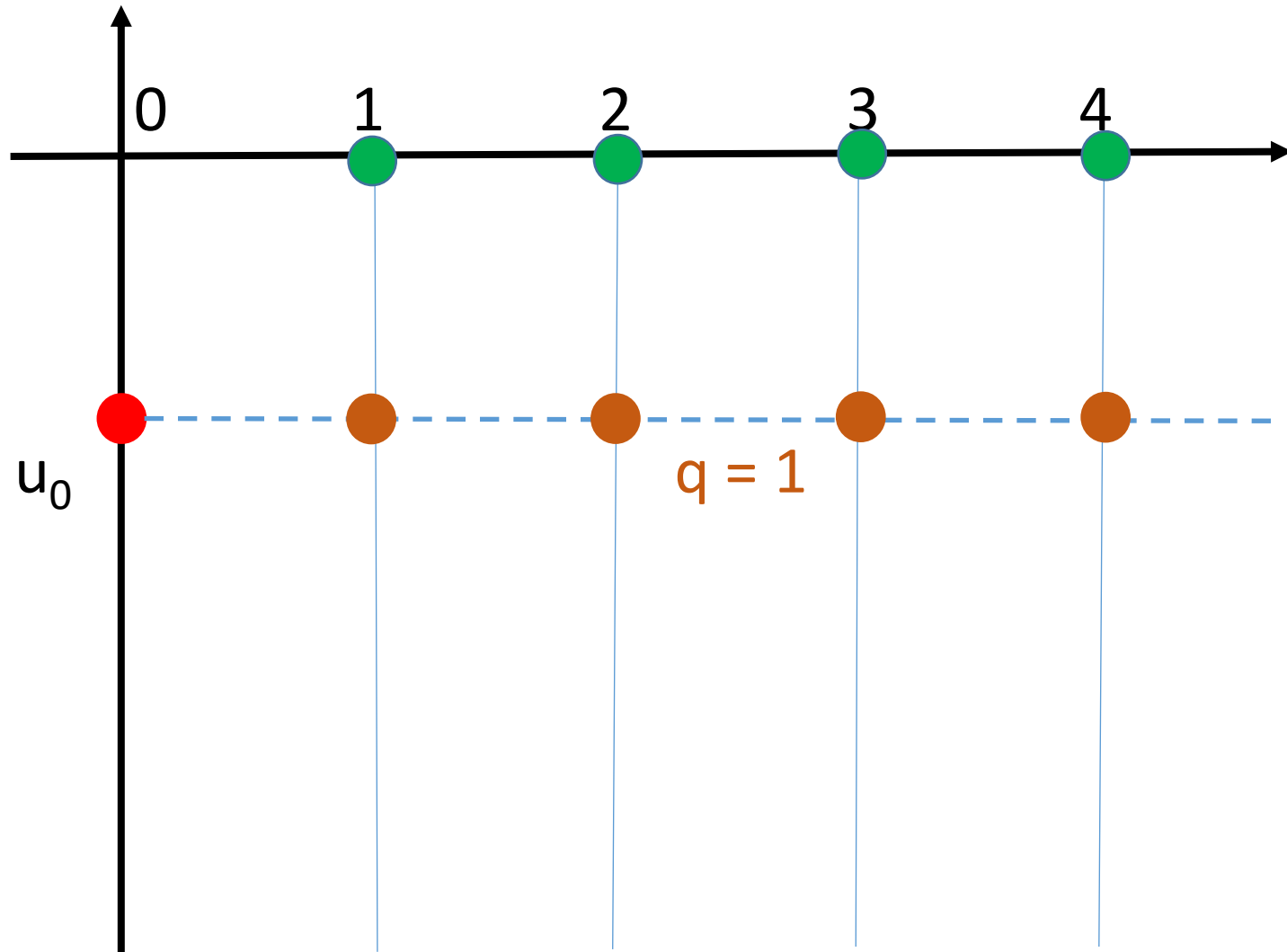
Suite constante sur $\mathbb{N}$

Limite : lorsque $n \rightarrow +\infty$ $u_n \rightarrow u_0$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

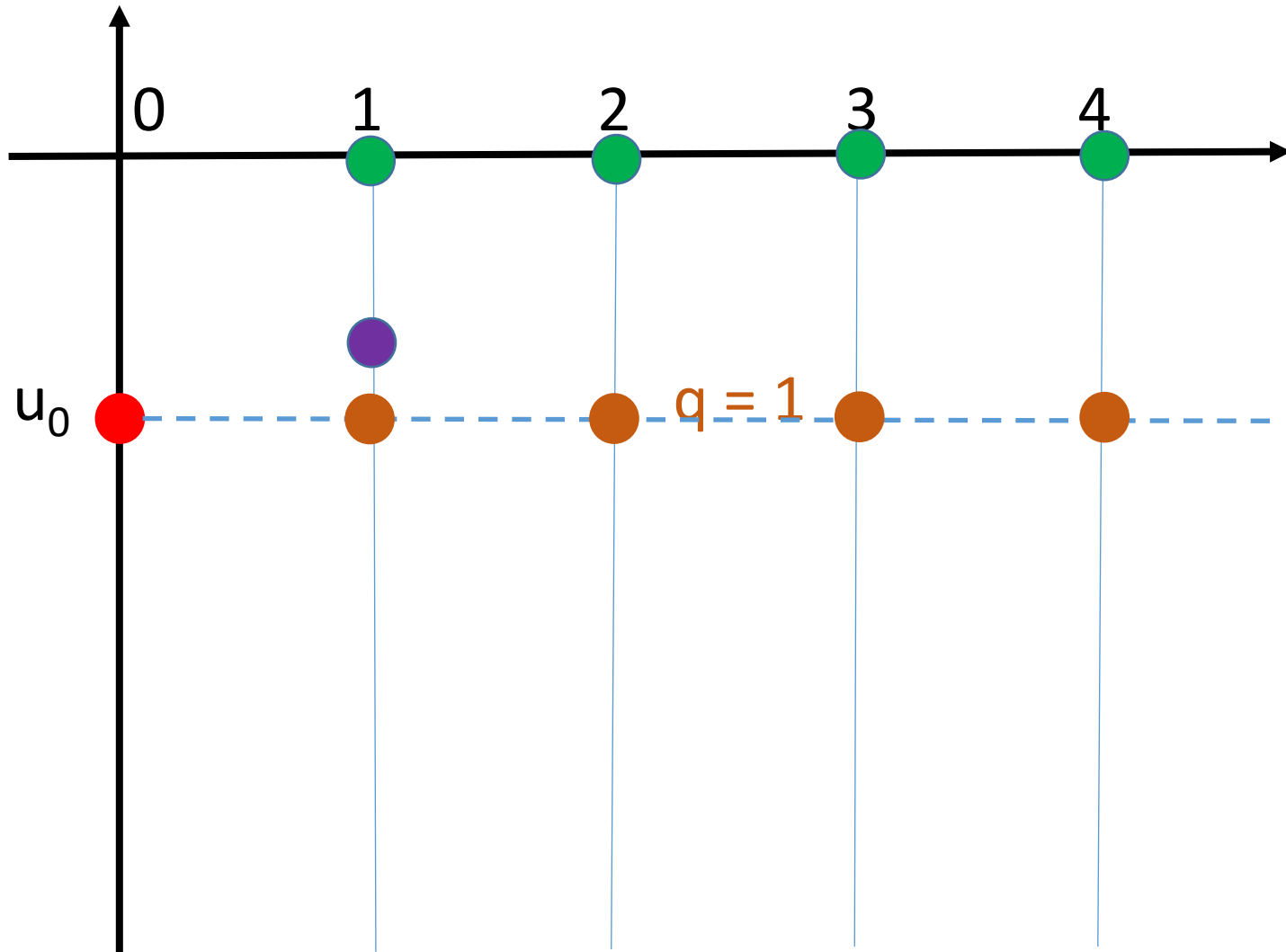
3^{ème} cas : $0 < q < 1$

par exemple,
tracé pour $q = 0,8$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



3^{ème} cas : $q = 0$

$$0 < q < 1$$

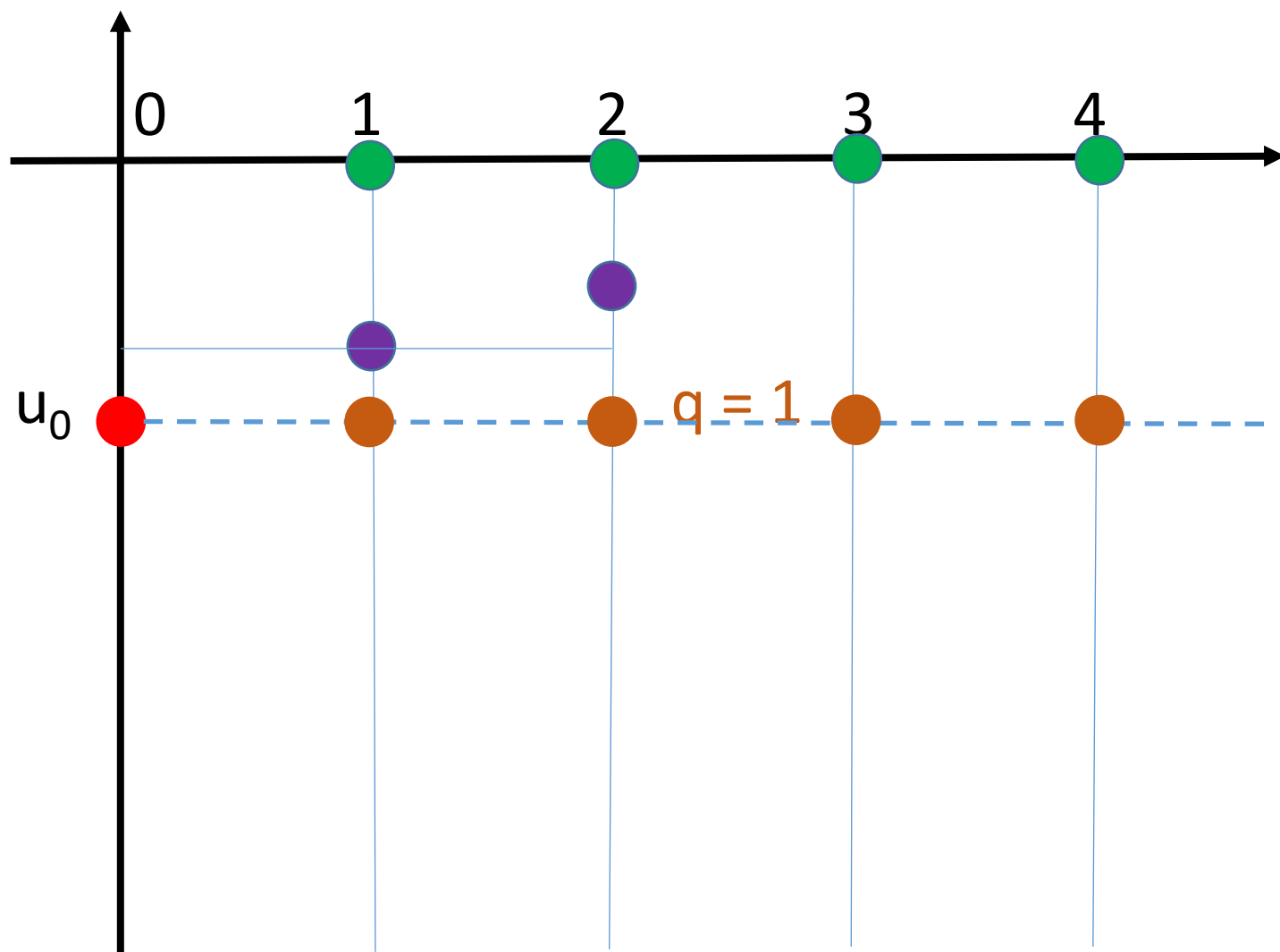
$$0 \times u_0 > q \times u_0 > 1 \times u_0$$

$$0 > u_1 > u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$0 \times u_0 > q \times u_0 > 1 \times u_0$$

$$0 > u_1 > u_0$$

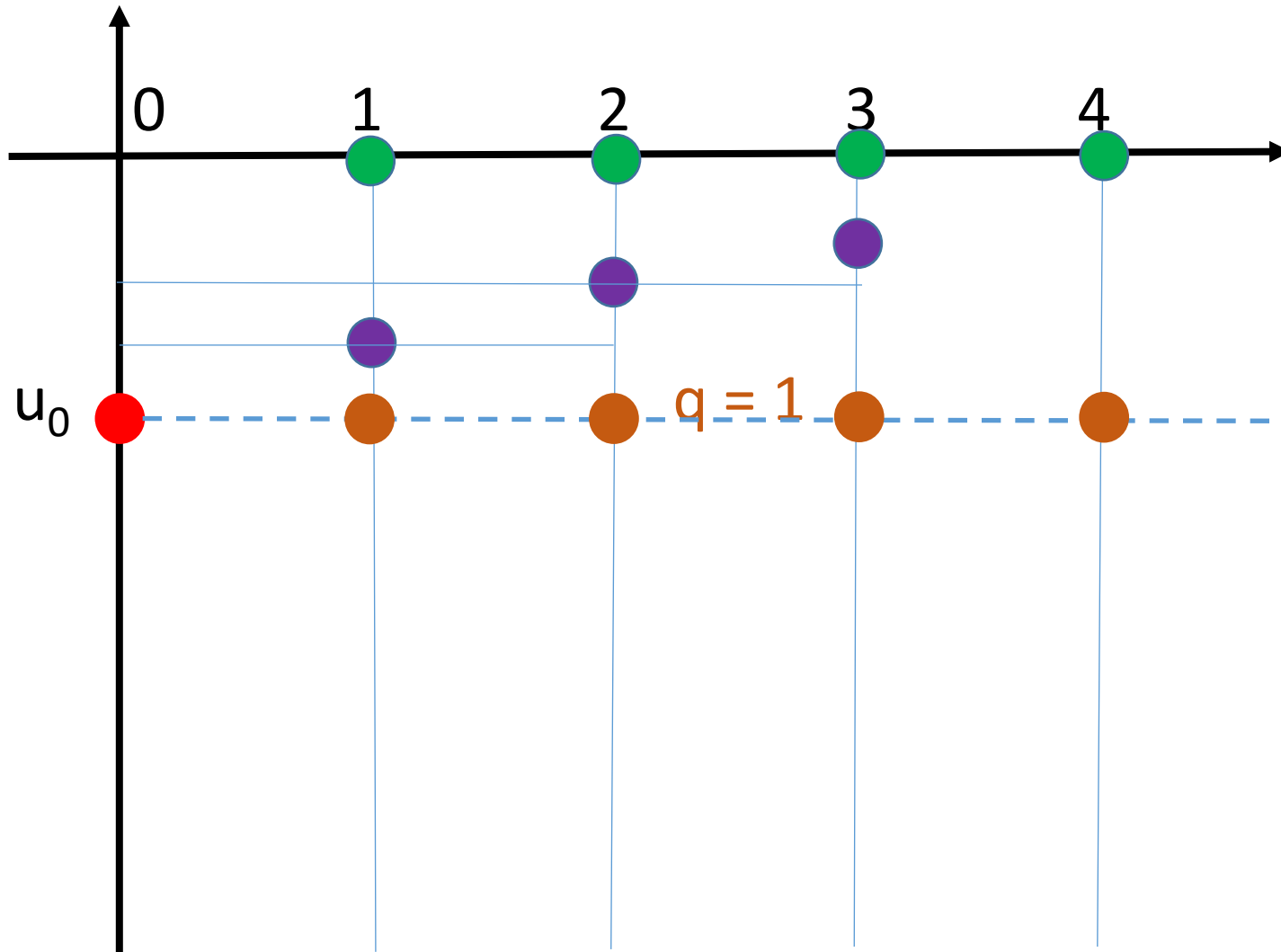
$$0 \times q > u_1 \times q > u_0 \times q$$

$$0 > u_2 > u_1$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$0 \times u_0 > q \times u_0 > 1 \times u_0$$

$$0 > u_1 > u_0$$

$$0 \times q > u_1 \times q > u_0 \times q$$

$$0 > u_2 > u_1$$

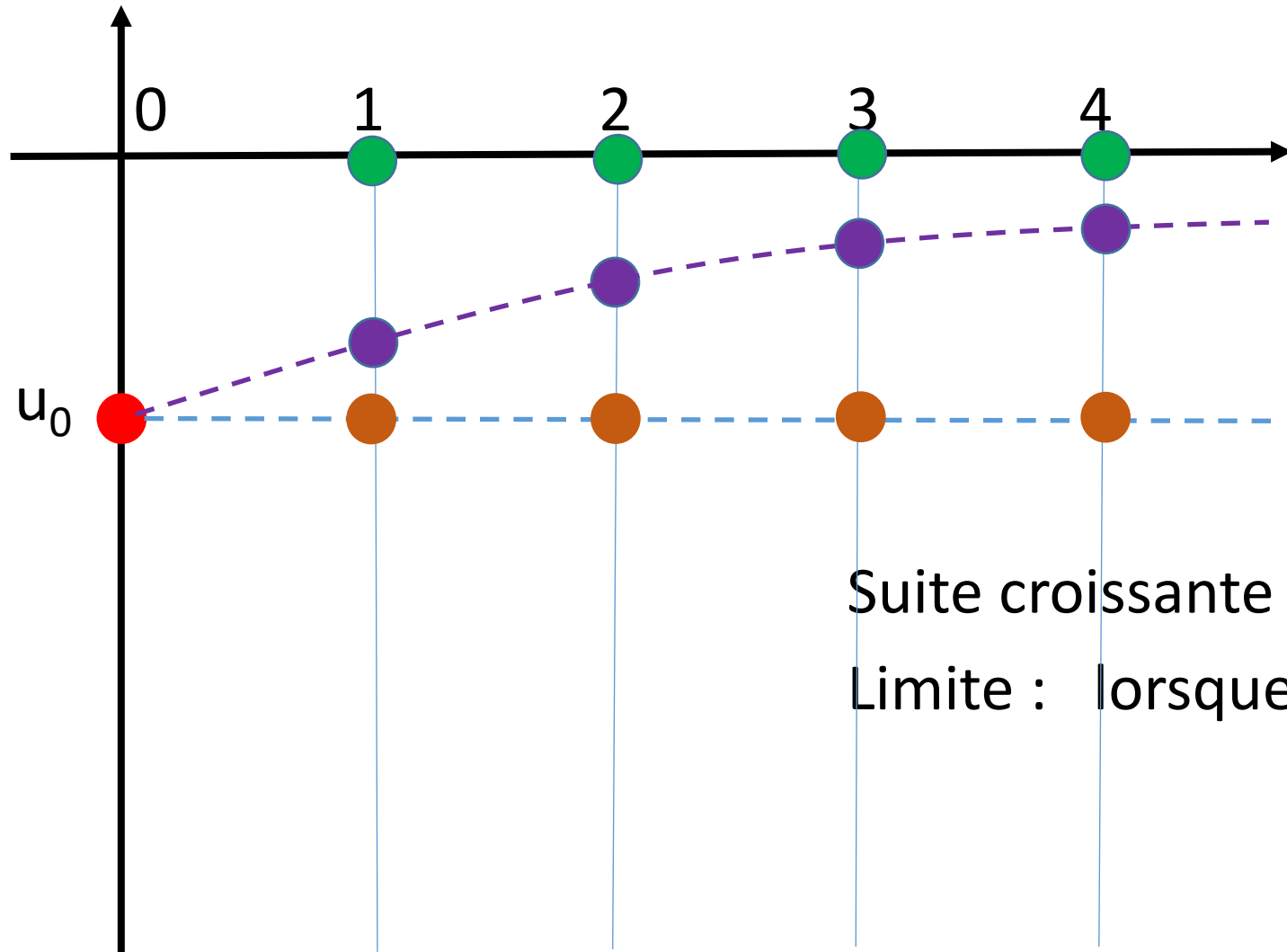
$$0 \times q > u_2 \times q > u_1 \times q$$

$$0 > u_3 > u_2$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

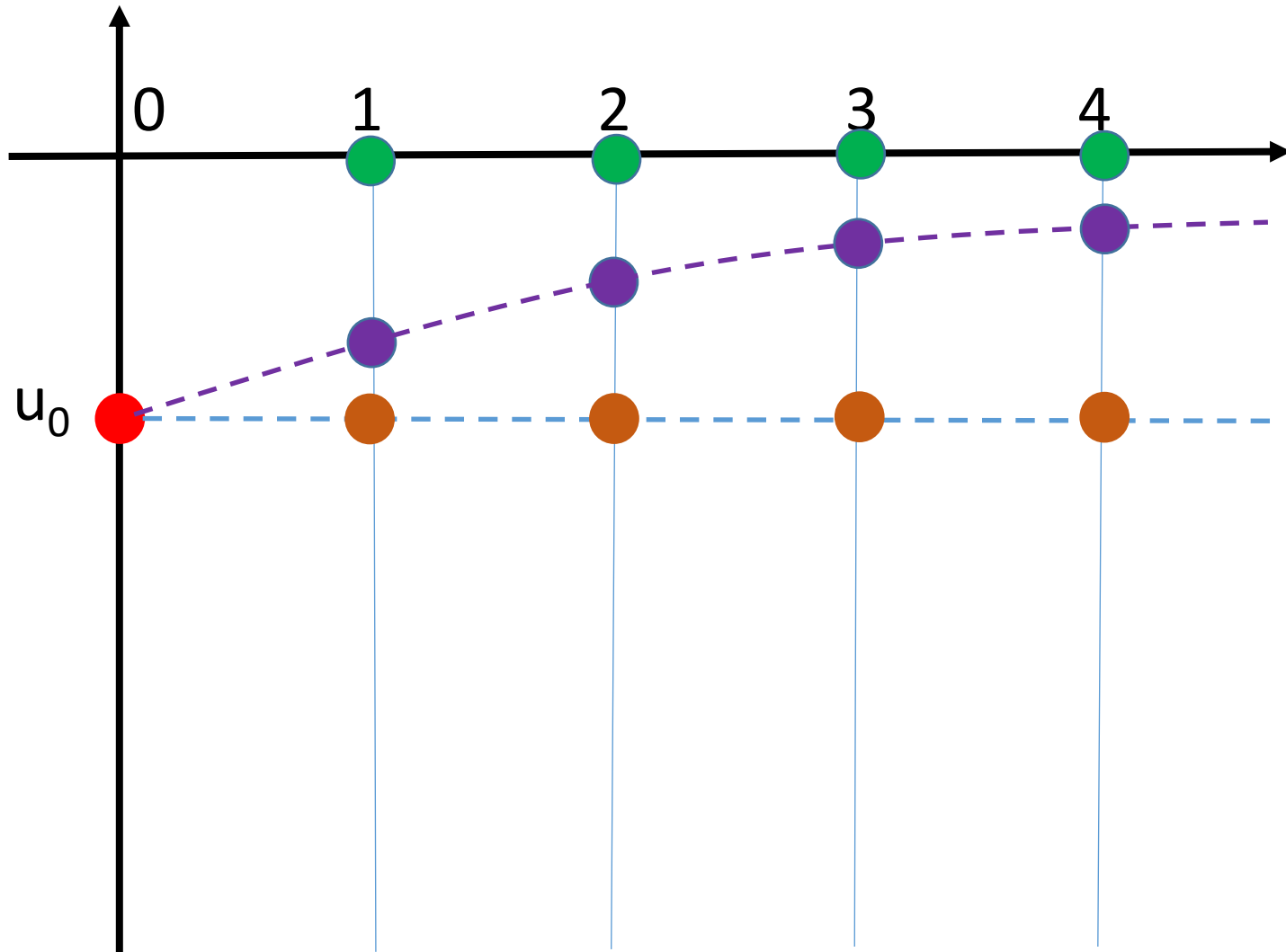
Suite croissante

Limite : lorsque $n \rightarrow +\infty$ $u_n \rightarrow 0$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

4^{ème} cas : $q > 1$

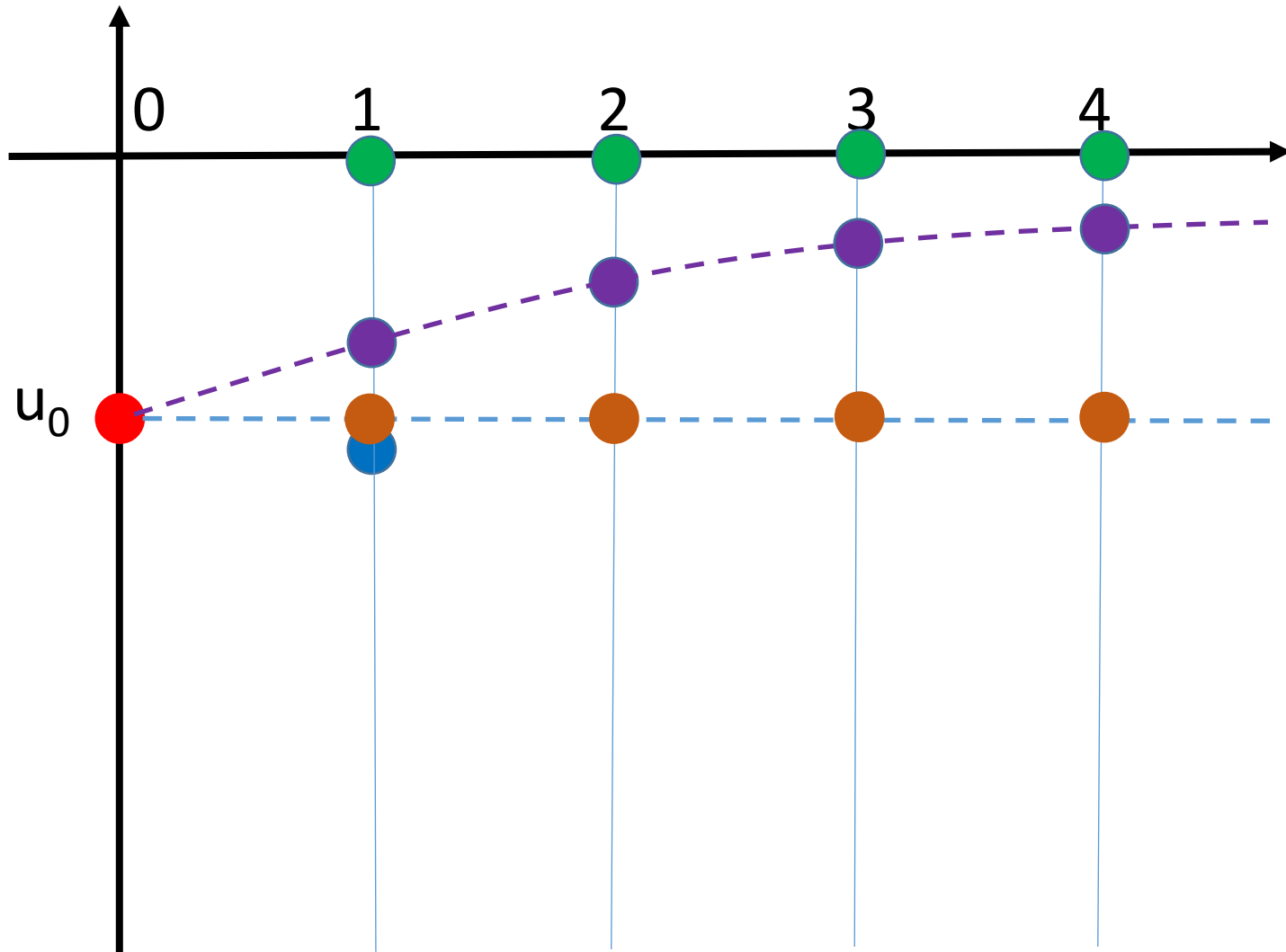
par exemple,

tracé pour $q = 1,2$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

4^{ème} cas : $q > 1$

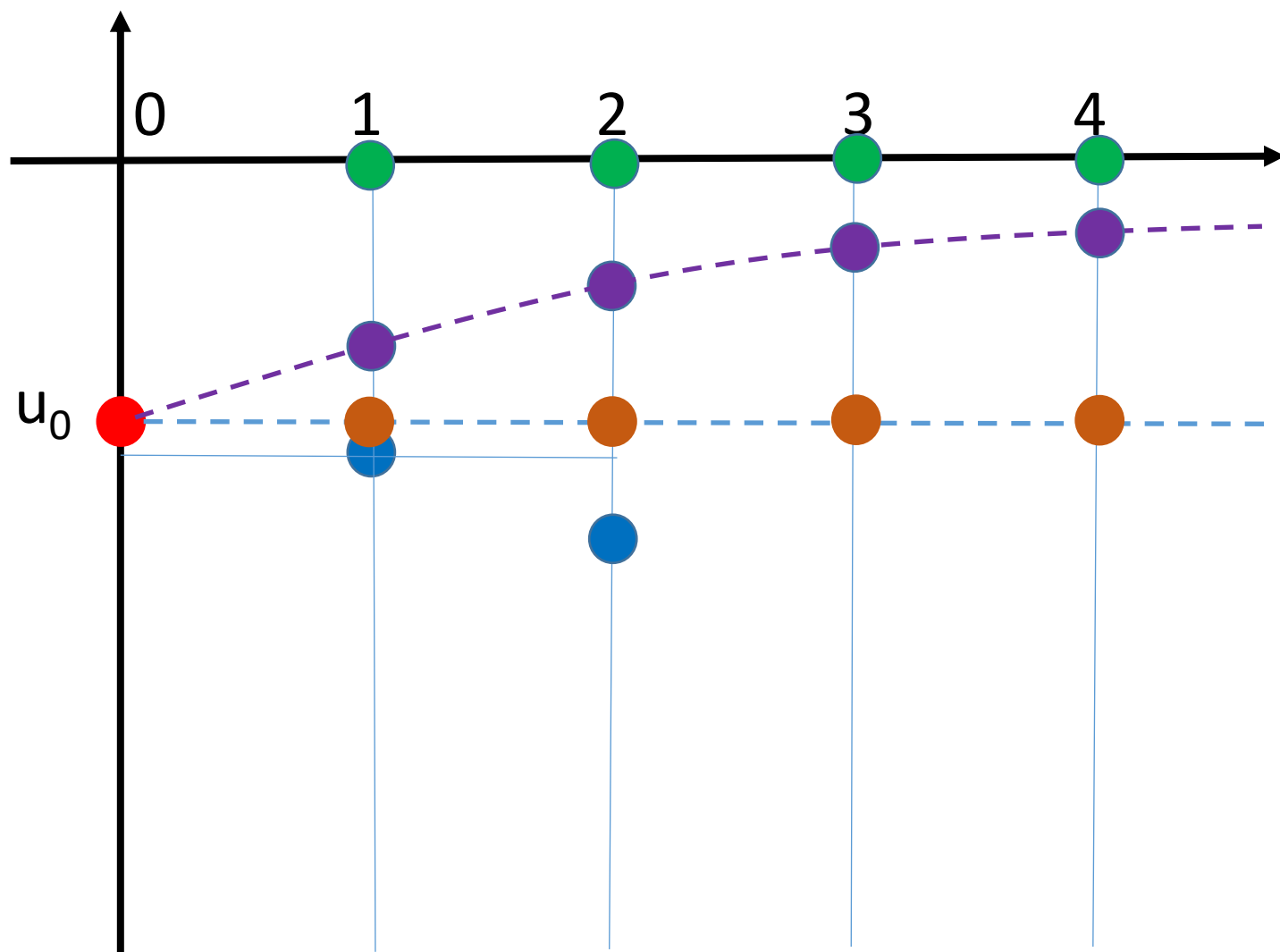
$$q \times u_0 > 1 \times u_0$$

$$u_1 < u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

$$q > 1$$

$$q \times u_0 > 1 \times u_0$$

$$u_1 < u_0$$

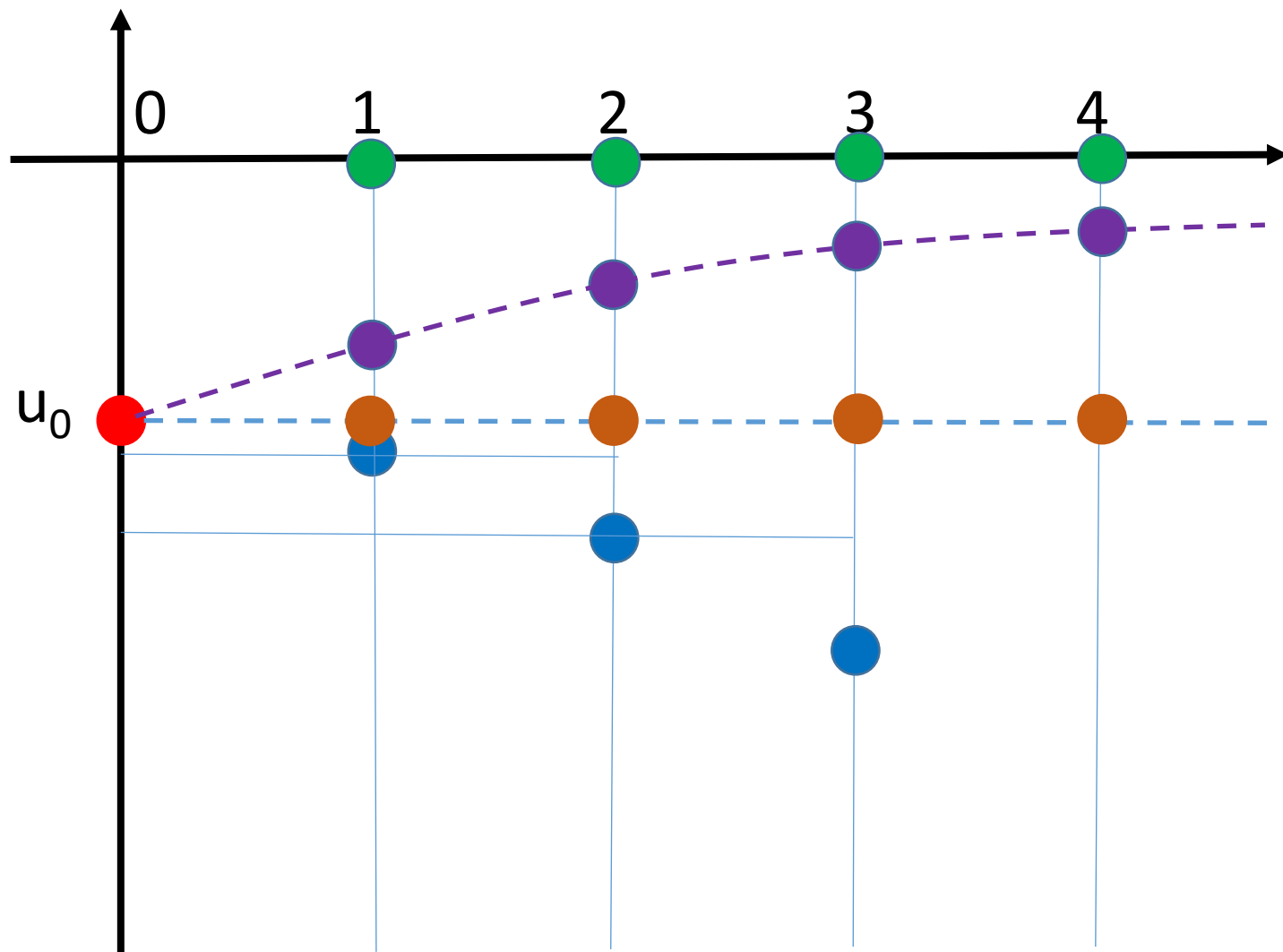
$$q \times u_1 < q \times u_0$$

$$u_2 < u_1$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

$$q > 1$$

$$q \times u_0 > 1 \times u_0$$

$$u_1 < u_0$$

$$q \times u_1 < q \times u_0$$

$$u_2 < u_1$$

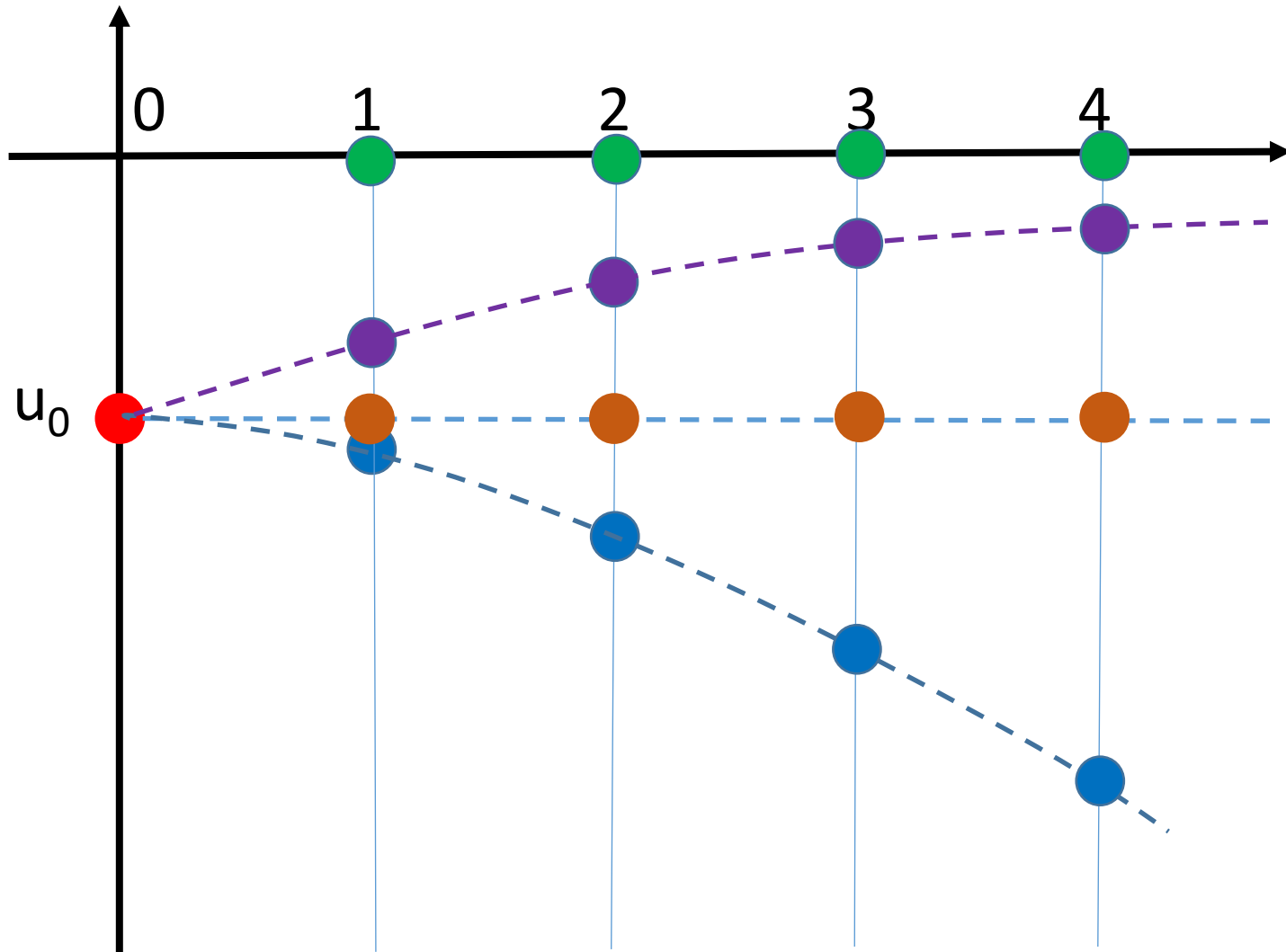
$$q \times u_2 < q \times u_1$$

$$u_3 < u_2$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

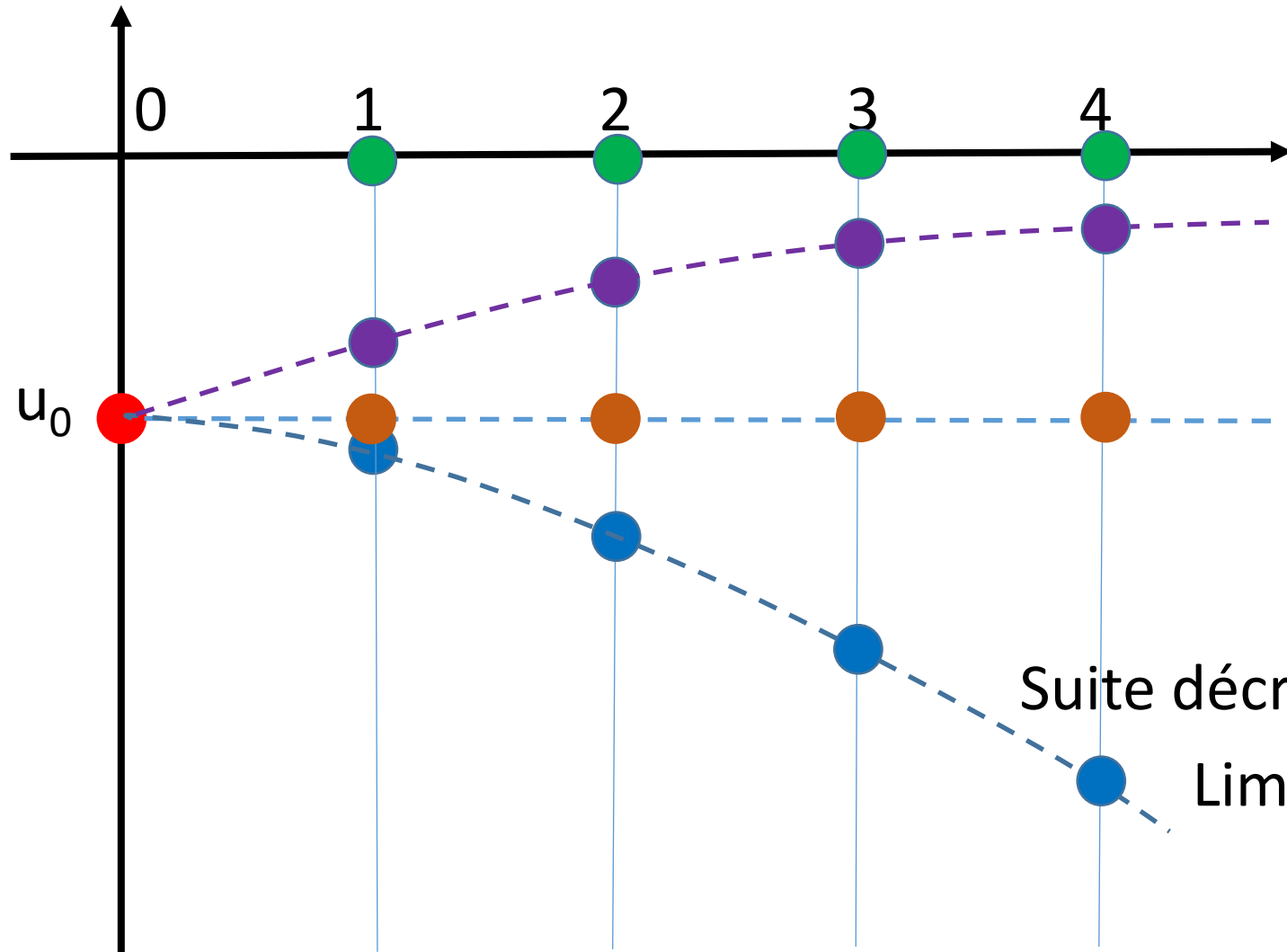
$$q = 1$$

$$q > 1$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

1^{er} cas $q \geq 0$



$$q = 0$$

$$0 < q < 1$$

$$q = 1$$

$$q > 1$$

Suite décroissante

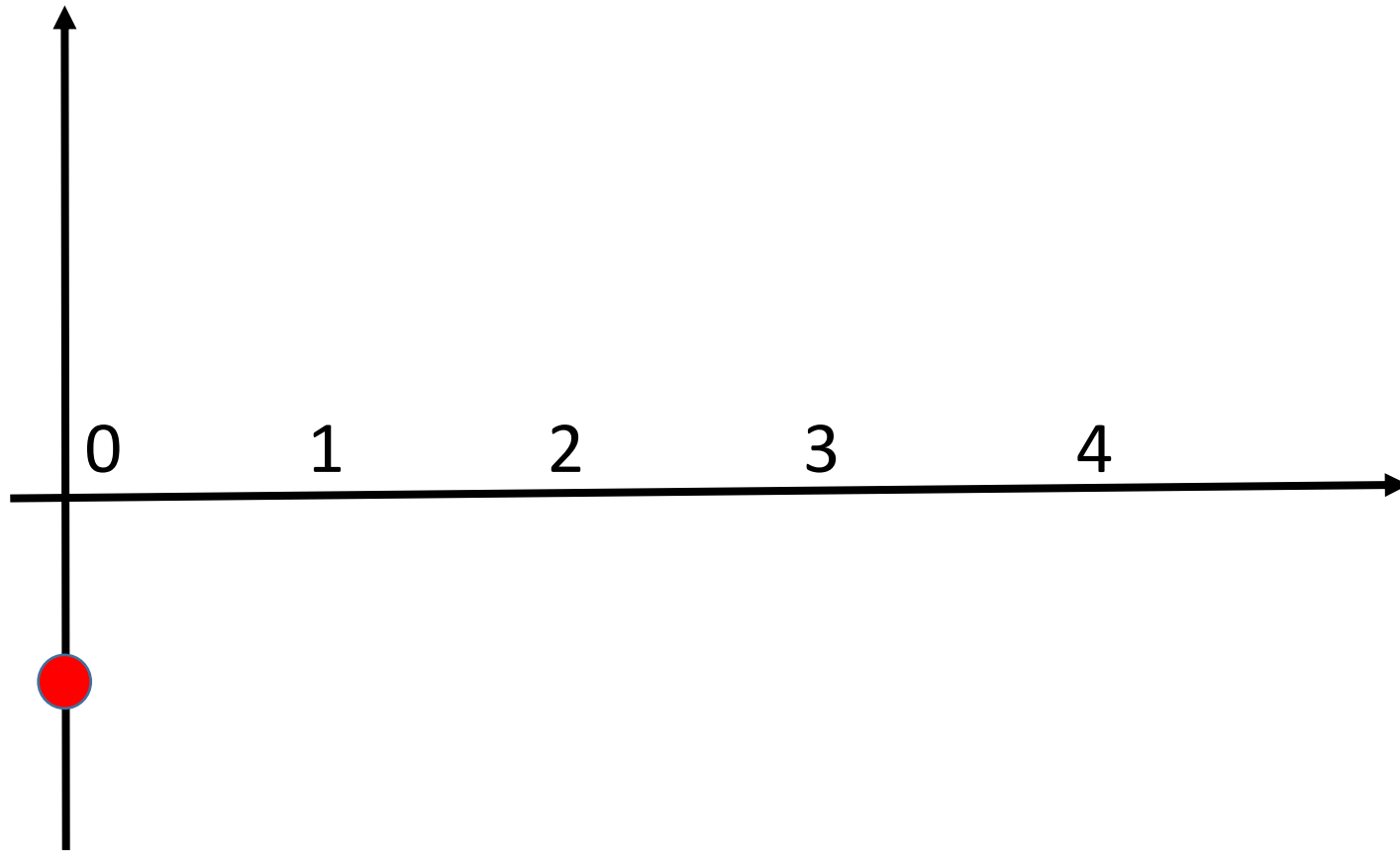
Limite : lorsque $n \rightarrow +\infty$

$$u_n \rightarrow -\infty$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



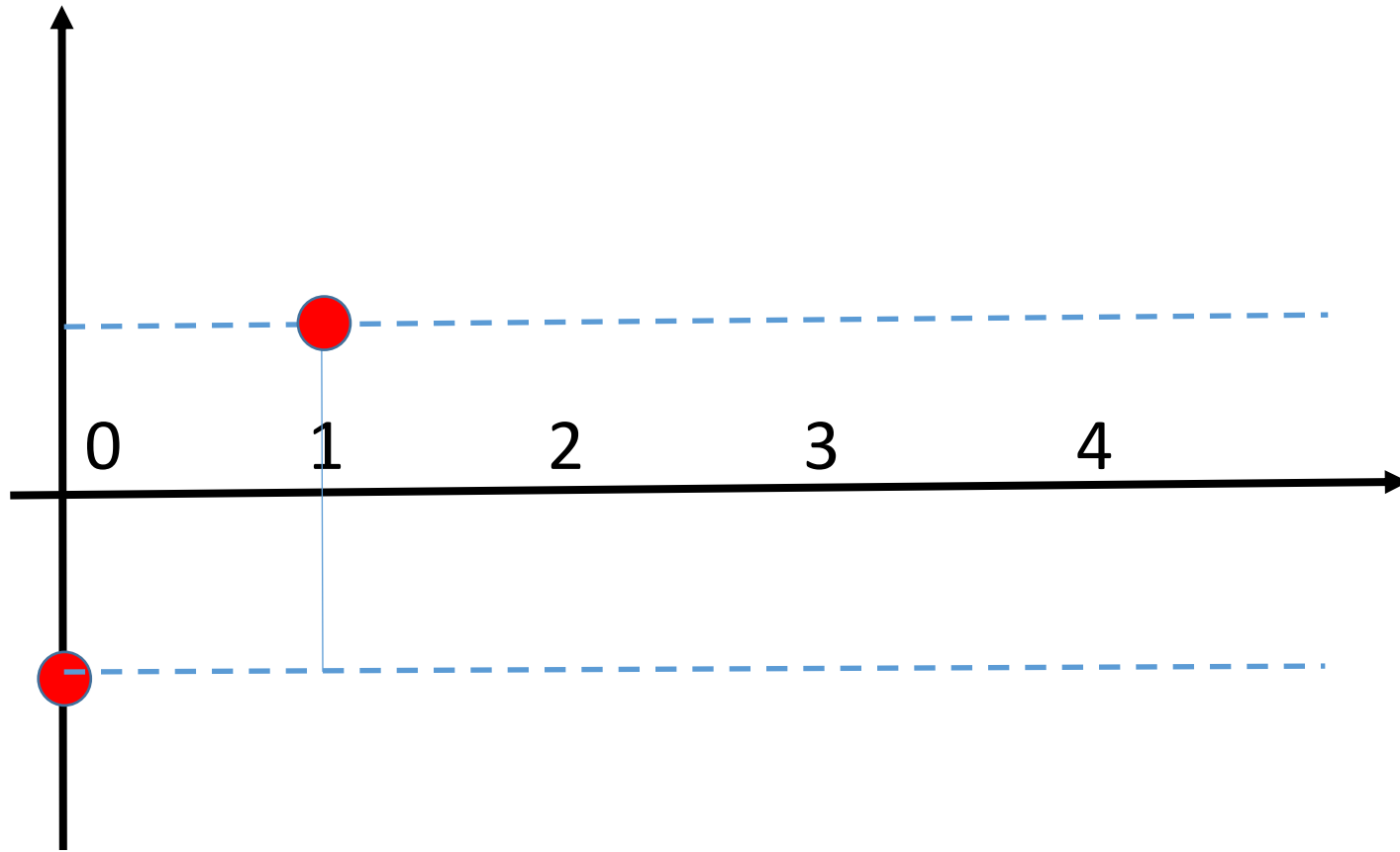
5^{ème} cas : $q = -1$

$$u_0 < 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

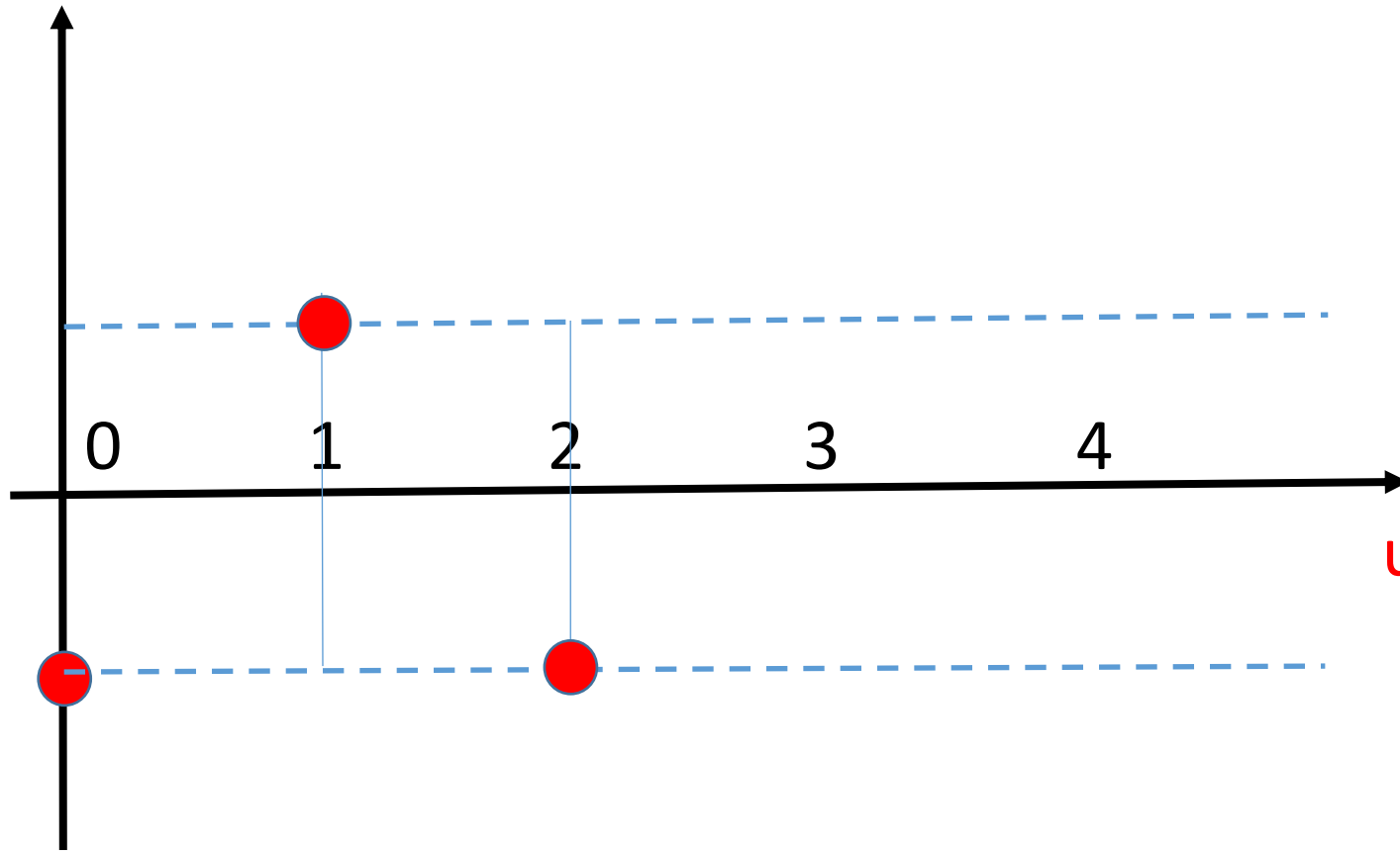
$$u_0 < 0$$

$$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$u_0 < 0$$

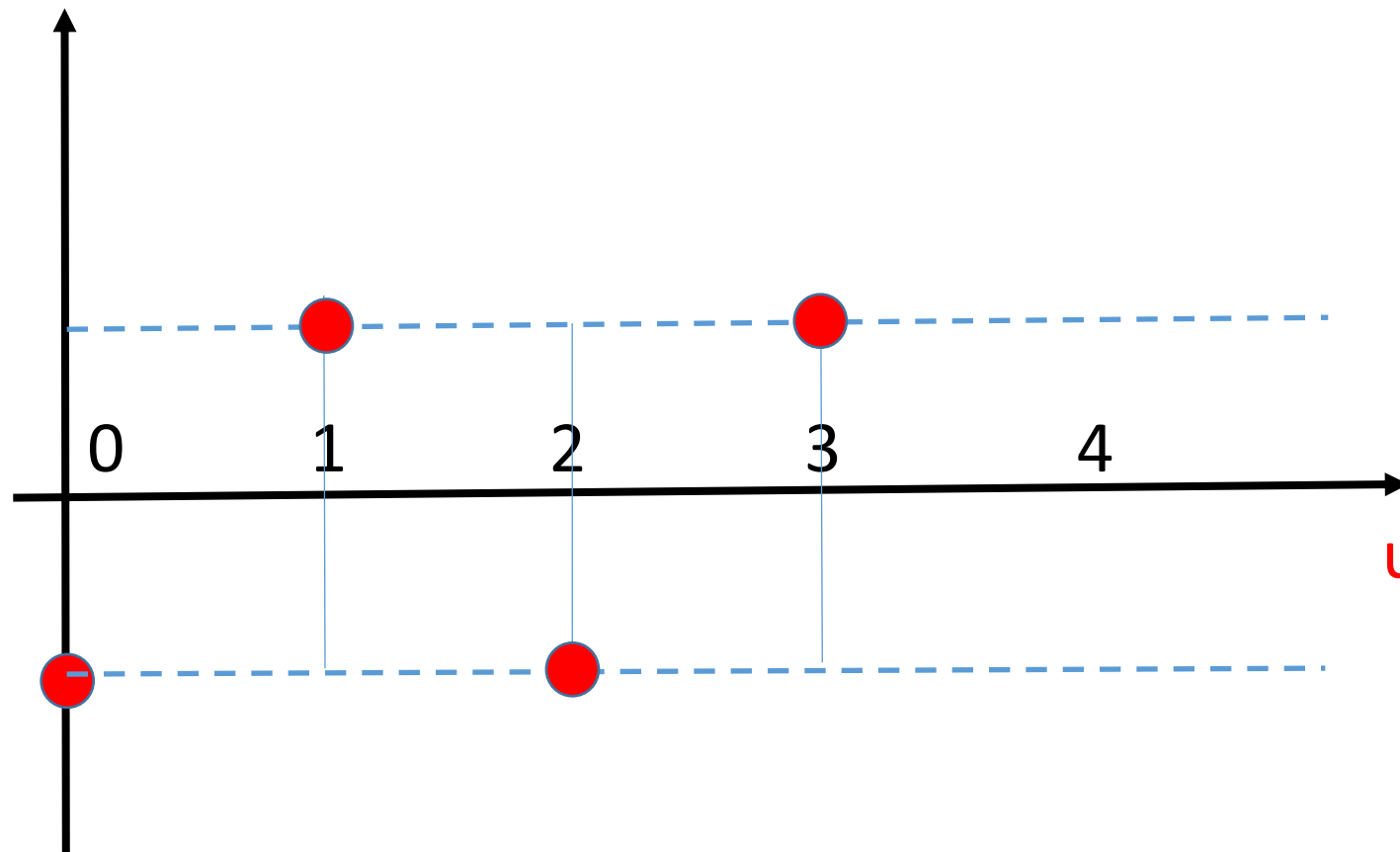
$$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$$

$$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$u_0 < 0$$

$$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$$

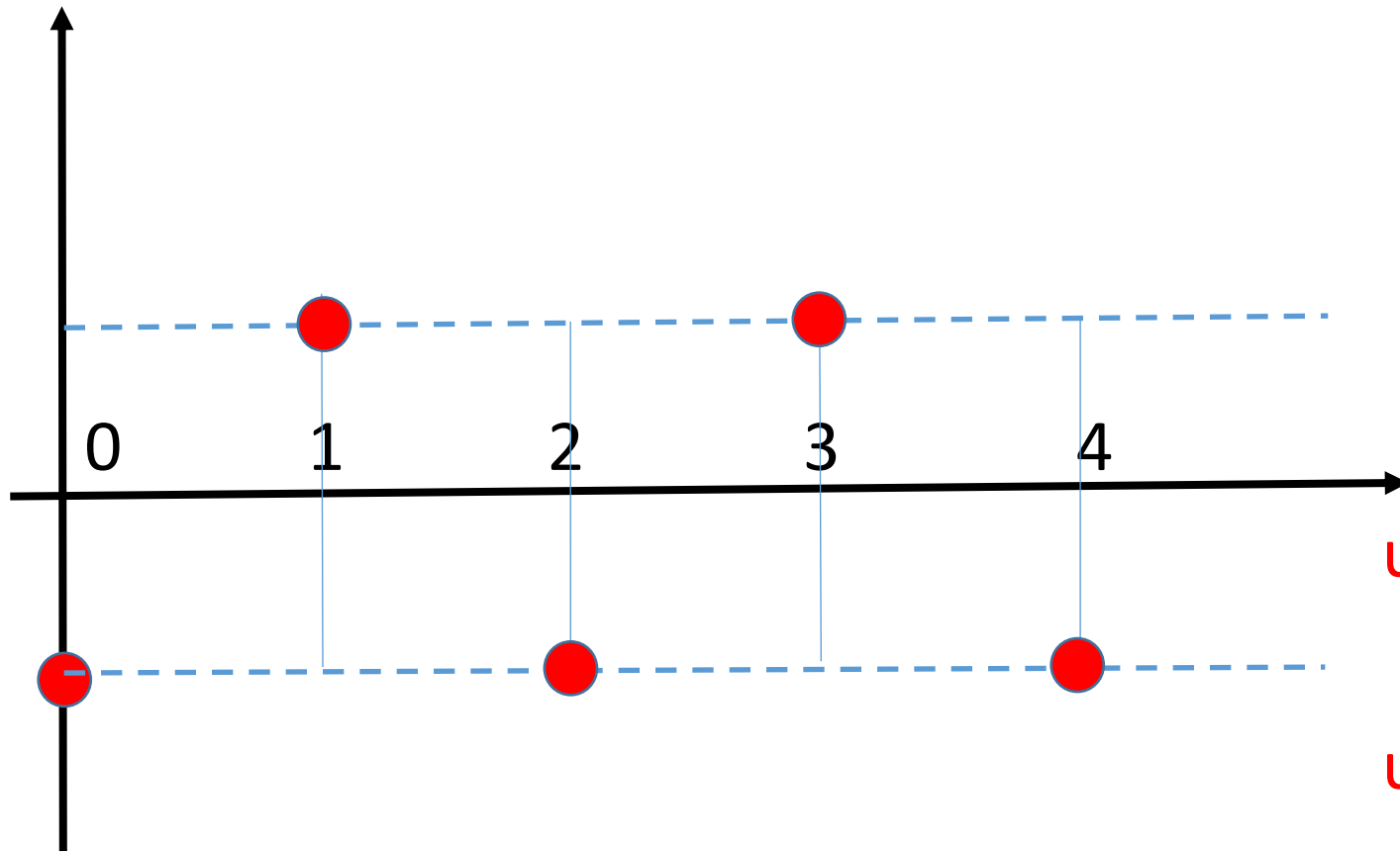
$$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$$

$$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$u_0 < 0$$

$$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$$

$$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$$

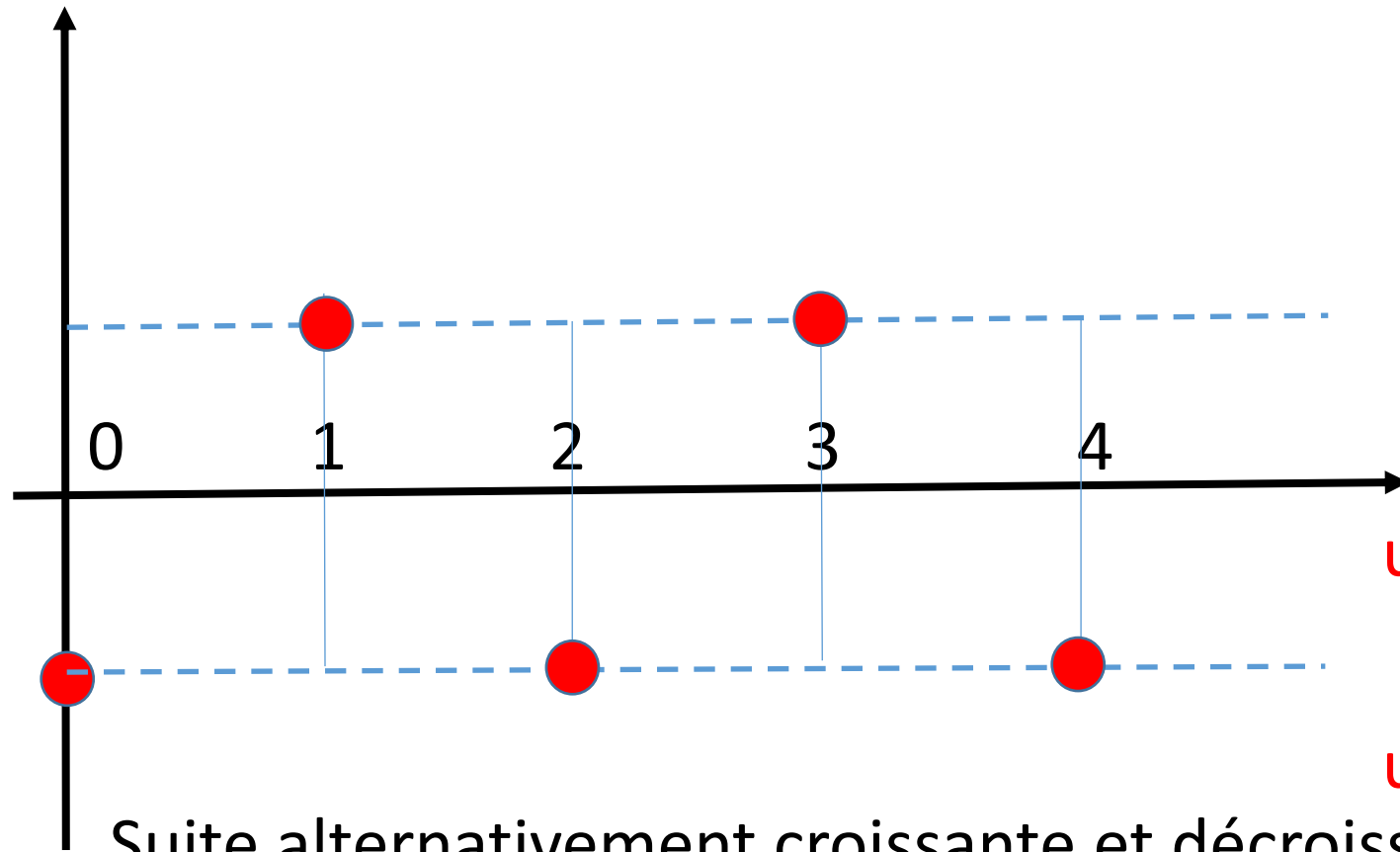
$$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$$

$$u_4 = q \times u_3 = (-1) \times (-u_0) = u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$u_0 < 0$$

$$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$$

$$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$$

$$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$$

$$u_4 = q \times u_3 = (-1) \times (-u_0) = u_0$$

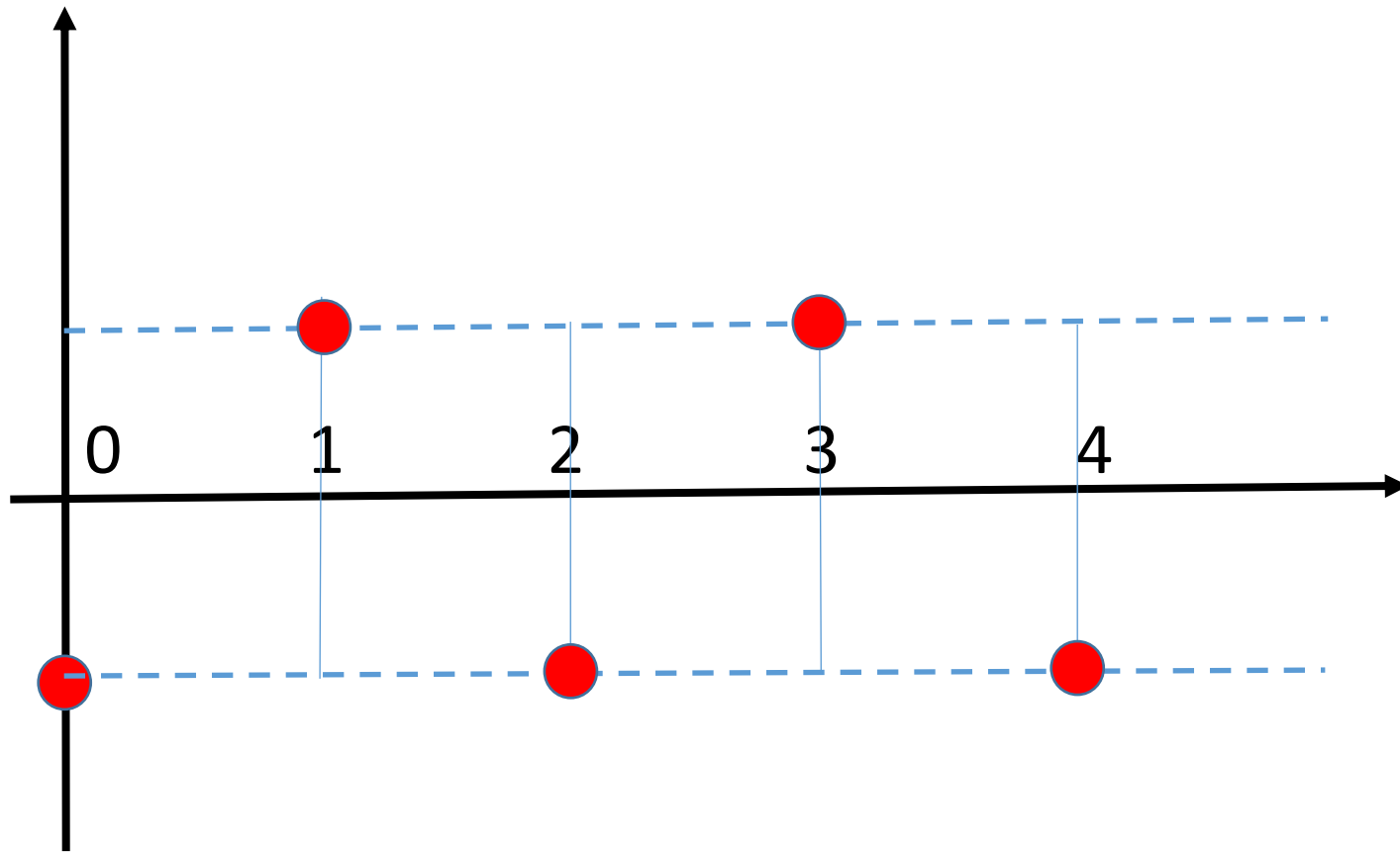
Suite alternativement croissante et décroissante

Pas de limite (les termes u_n ne se stabilisent pas).

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

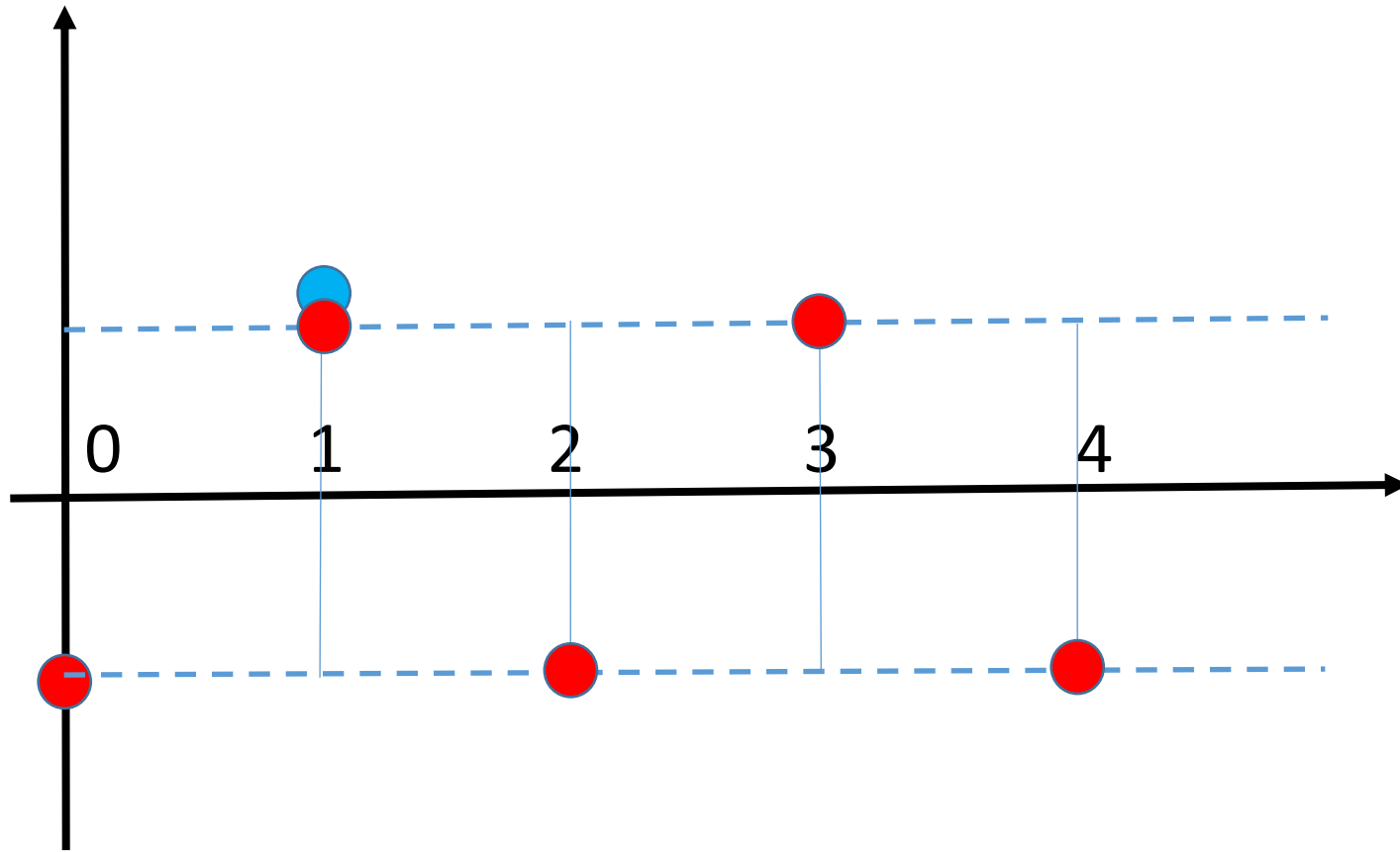
6^{ème} cas : $q < -1$

par ex. tracé pour $q = -1,1$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$q < -1$$

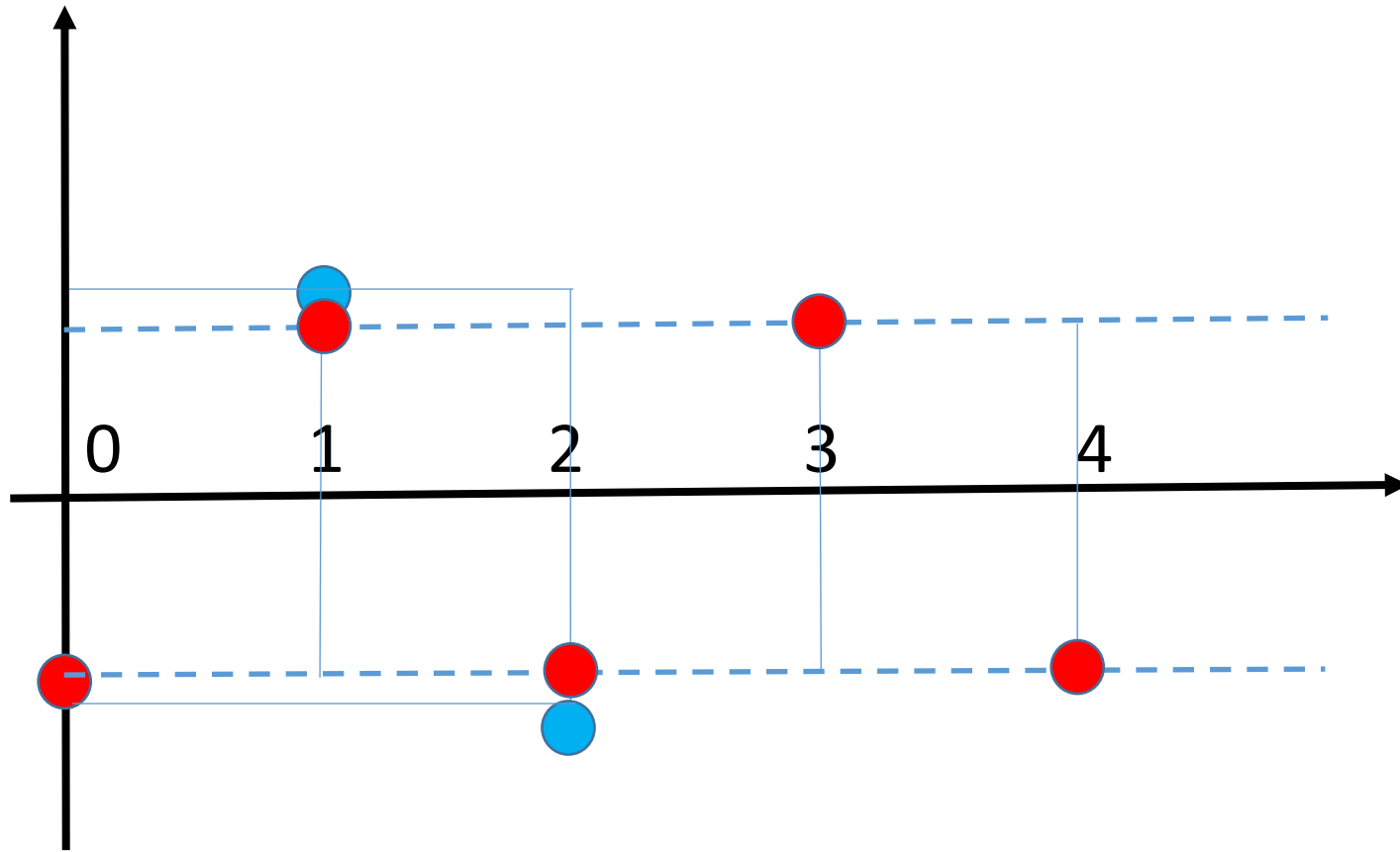
$$q \times u_0 > -1 \times u_0$$

$$u_1 > -u_0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$q < -1$$

$$q \times u_0 > -1 \times u_0$$

$$u_1 > -u_0$$

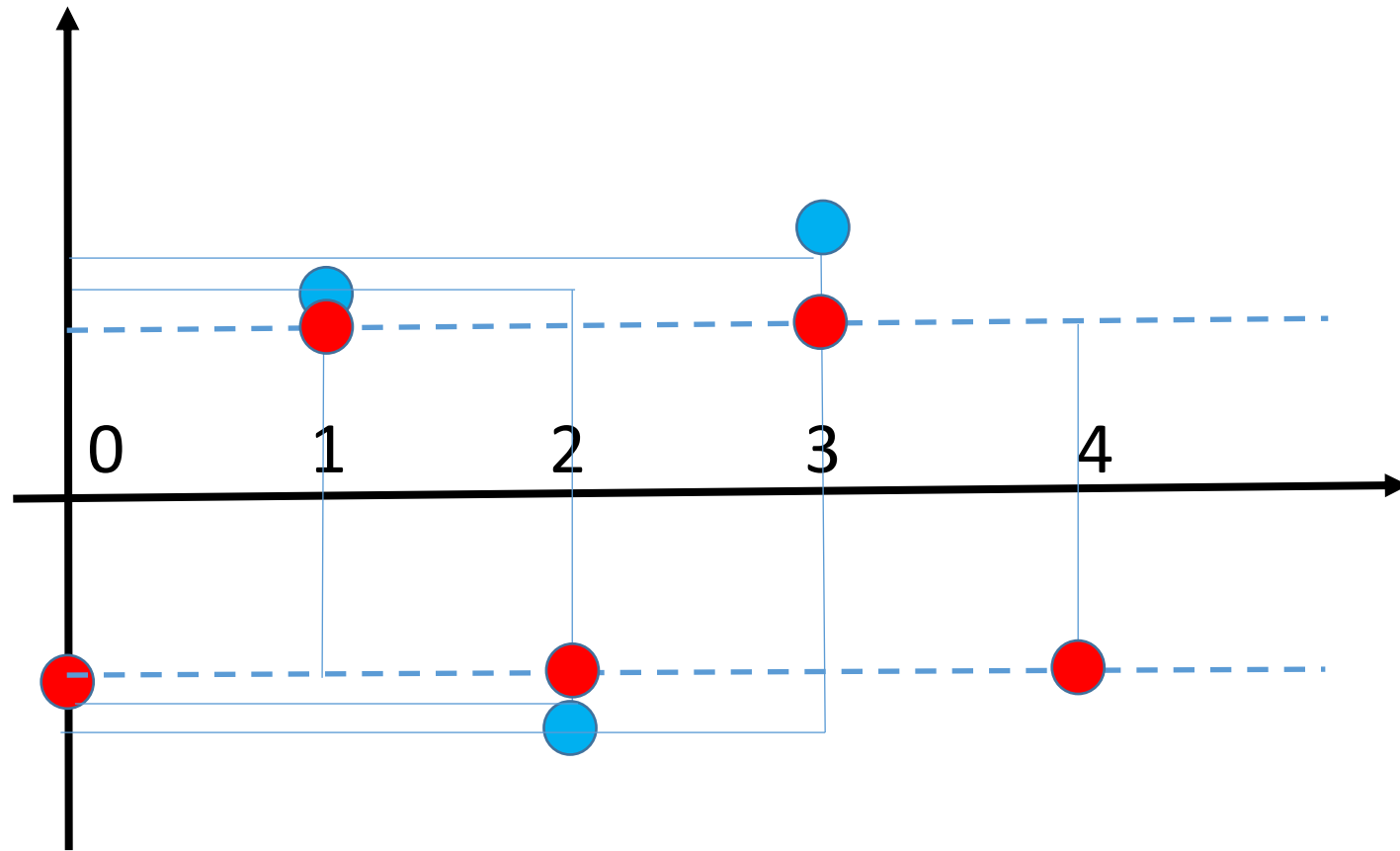
$$q \times u_1 < q \times (-u_0)$$

$$u_2 < -u_1$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q = -1$$

$$q < -1$$

$$q \times u_0 > -1 \times u_0$$

$$u_1 > -u_0$$

$$q \times u_1 < q \times (-u_0)$$

$$u_2 < -u_1$$

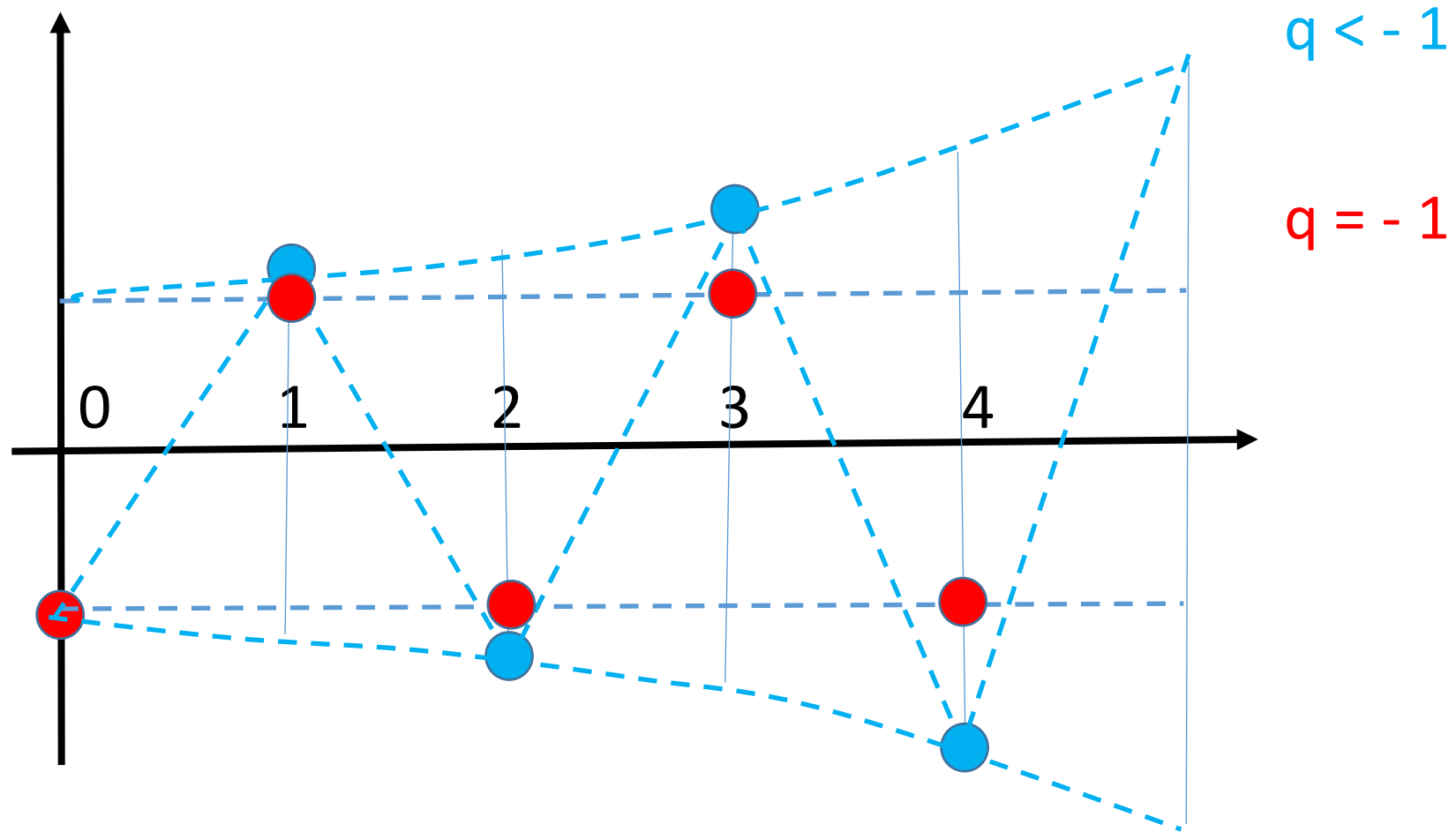
$$q \times u_2 > q \times (-u_1)$$

$$u_3 > -u_2$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

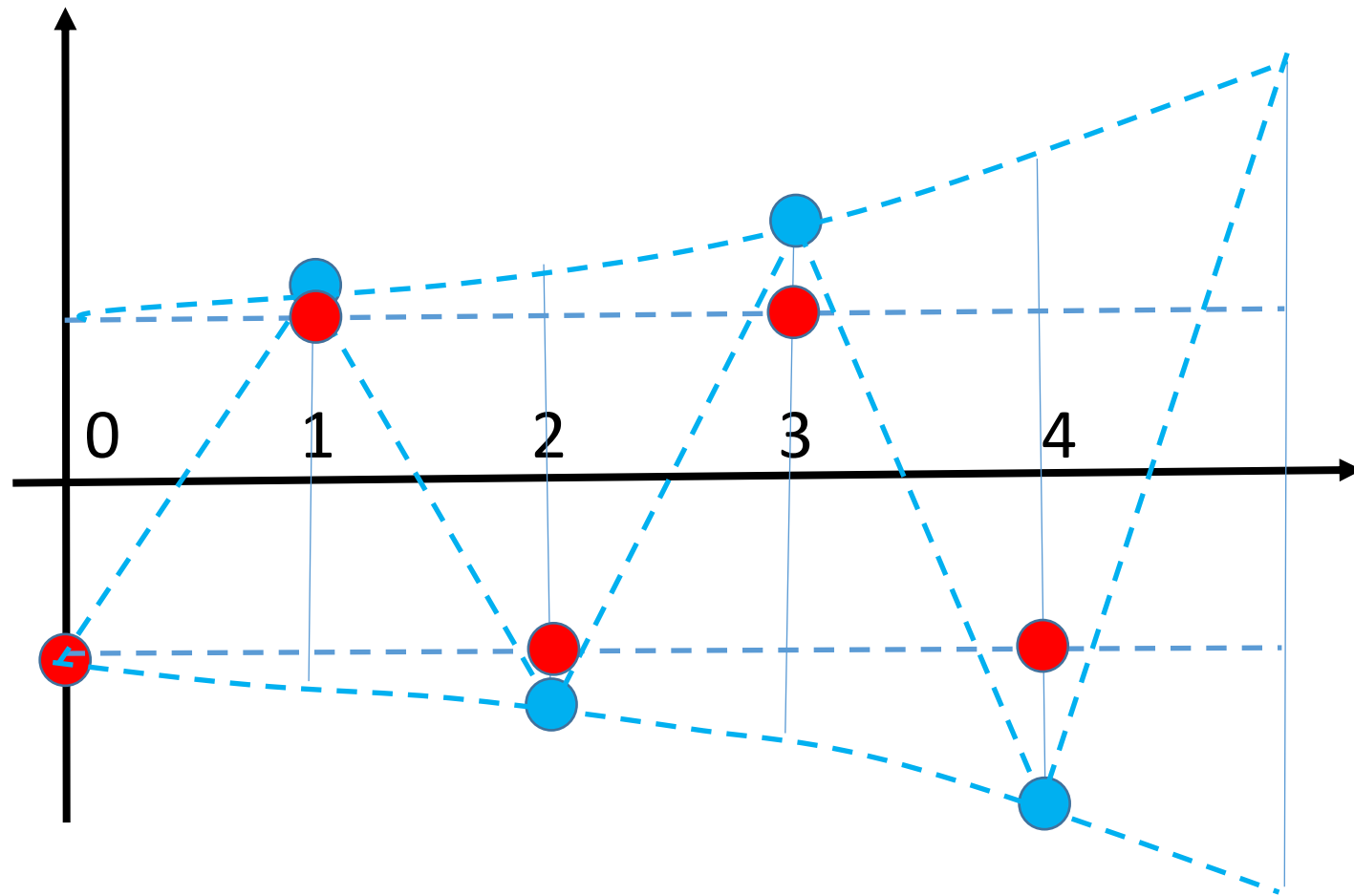
2^{ème} cas $q < 0$



$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$q < -1$

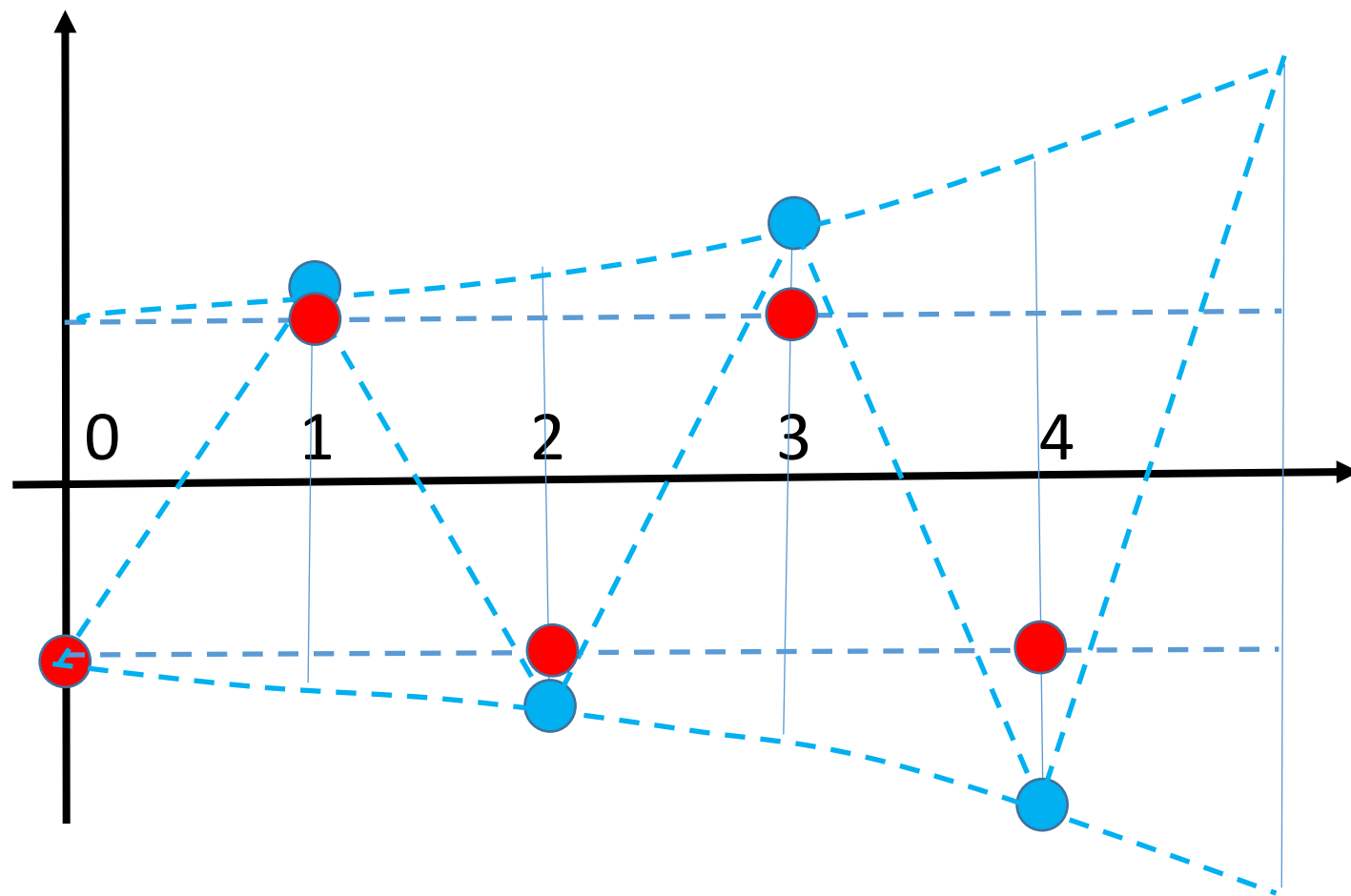
$q = -1$

Suite alternativement
croissante et décroissante
Pas de limite (les termes
 u_n ne se stabilisent pas)

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q < -1$$

$$q = -1$$

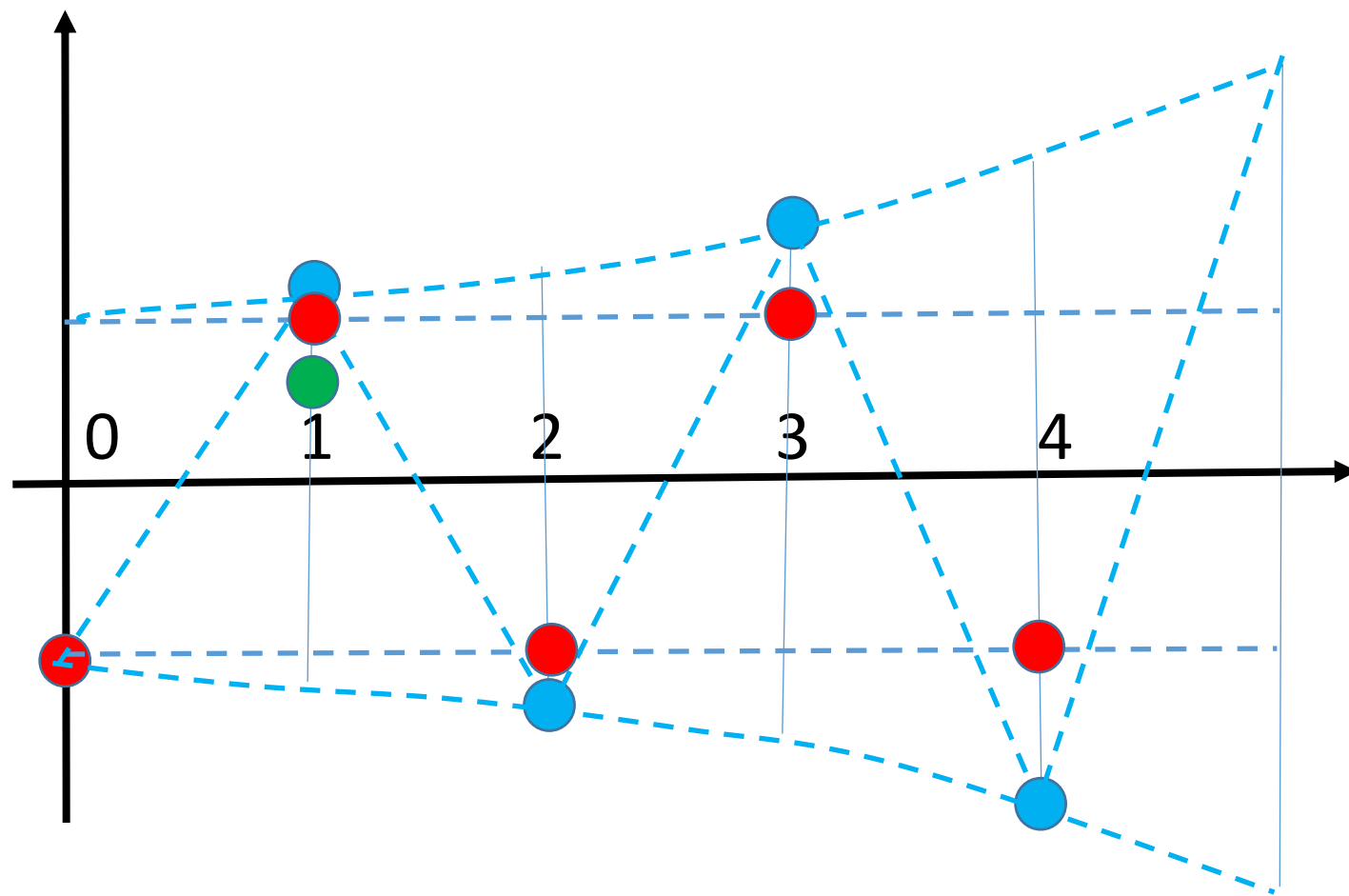
7^{ème} cas : $-1 < q < 0$

Par ex. tracé pour $q = -0,7$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q < -1$$

$$q = -1$$

$$-1 < q < 0$$

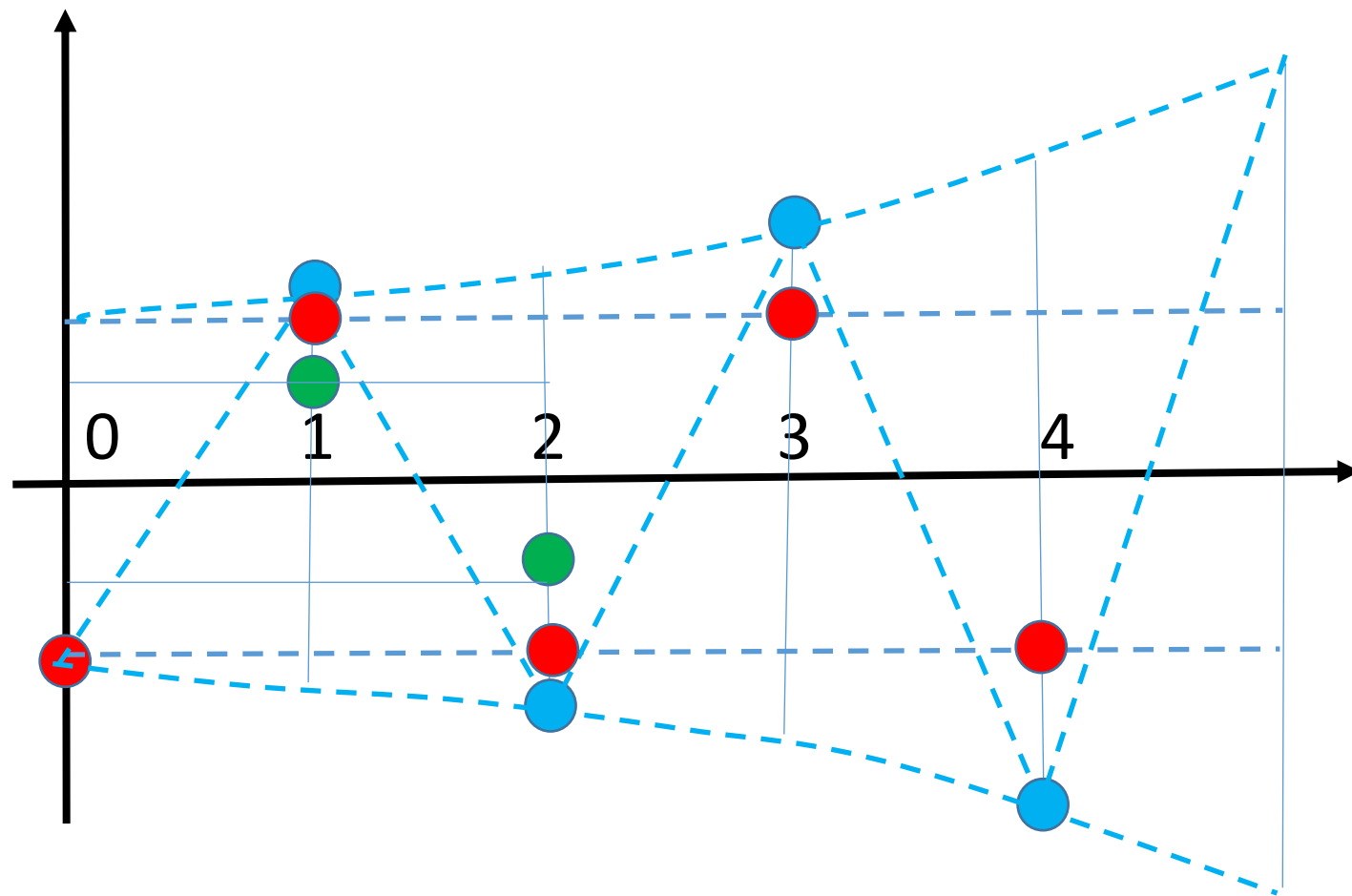
$$-1 \times u_0 > q \times u_0 > 0 \times u_0$$

$$-u_0 > u_1 > 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$$q < -1$$

$$q = -1$$

$$-1 < q < 0$$

$$-1 \times u_0 > q \times u_0 > 0 \times u_0$$

$$-u_0 > u_1 > 0$$

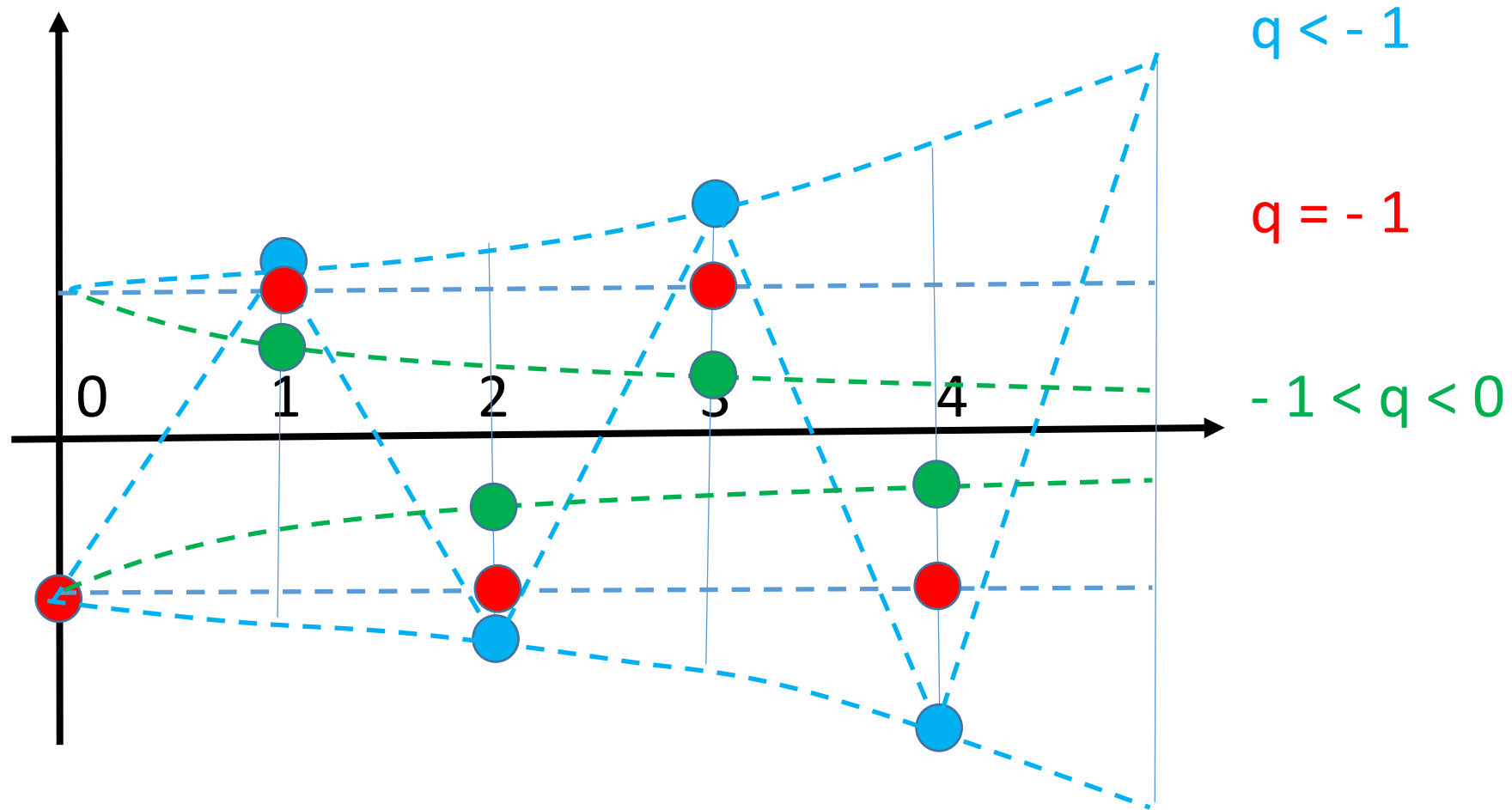
$$-u_0 \times q < u_1 \times q < 0 \times q$$

$$-u_1 < u_2 < 0$$

$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

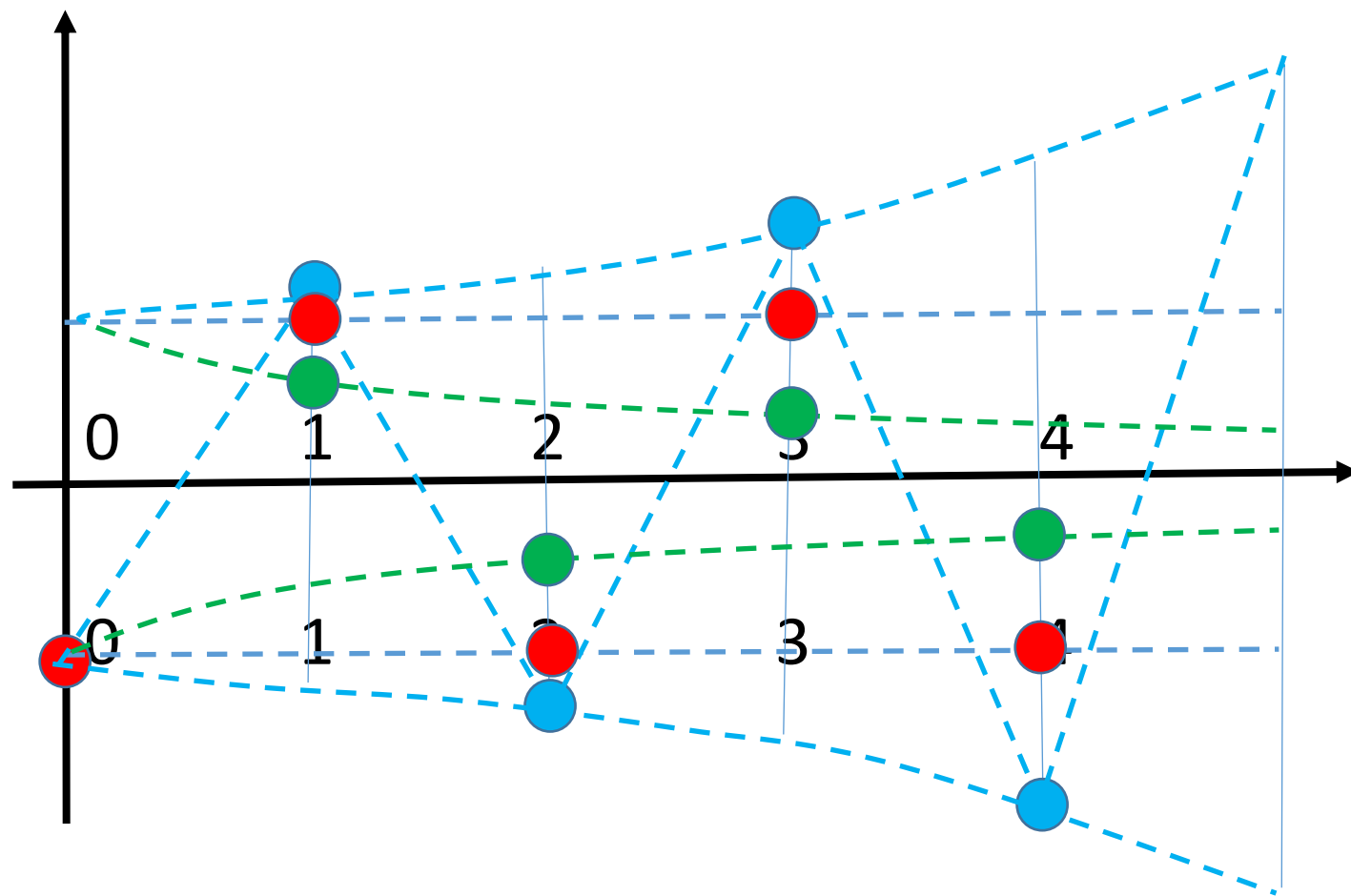
2^{ème} cas $q < 0$



$$u_{n+1} = u_n \times q$$

$$u_0 < 0$$

2^{ème} cas $q < 0$



$q < -1$

$q = -1$

$-1 < q < 0$

Suite alternativement
croissante et décroissante

Limite : lorsque $n \rightarrow +\infty$

$$u_n \rightarrow 0$$