# Exercice 12 :

Soient les suites $(u_n)$ définies par

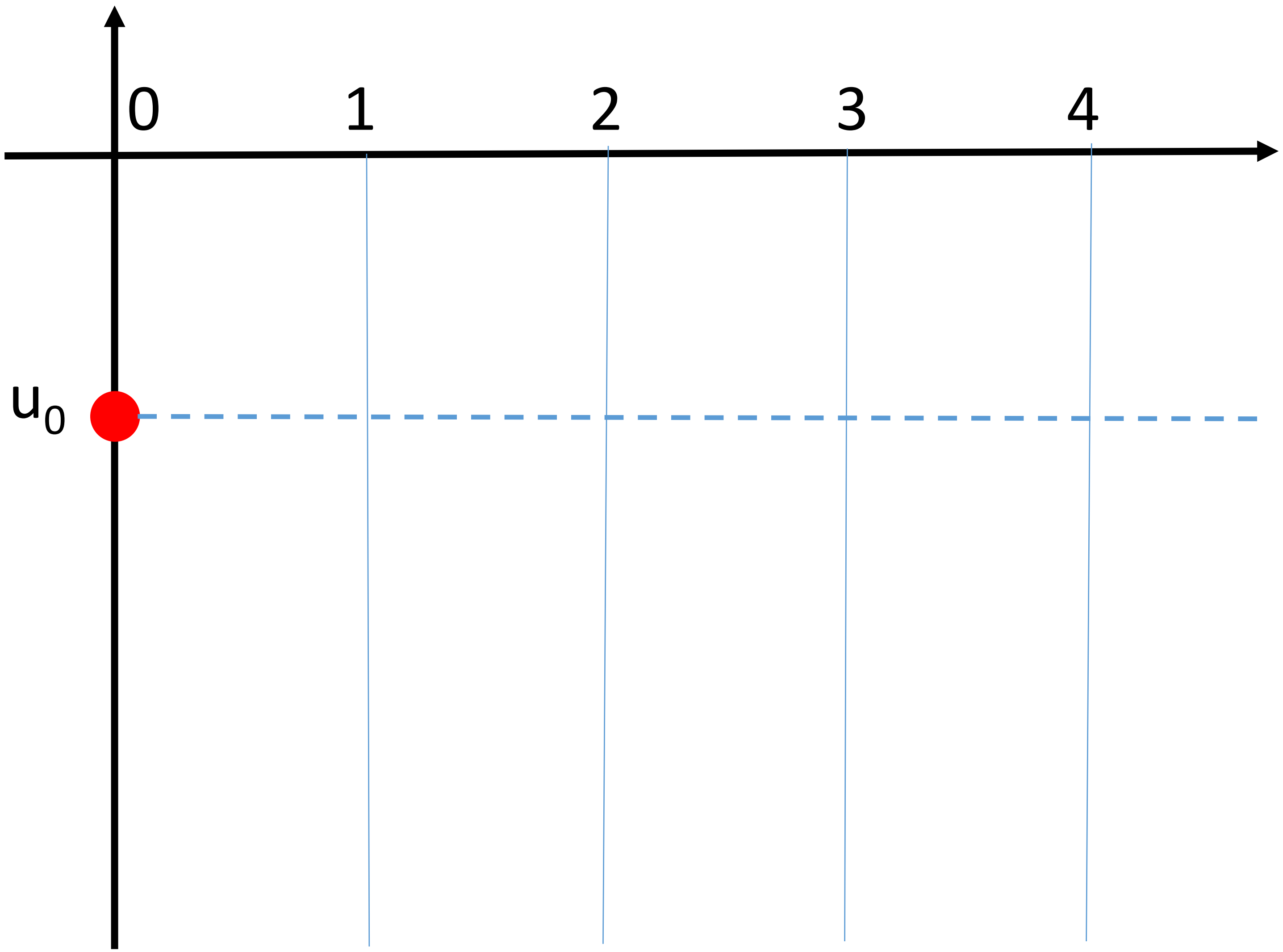
$u_{n+1} = u_n \times q$ et $u_0 < 0$

1°) Tracez ( les 5 premiers points et sur deux repères différents pour $q \geq 0$ et $q < 0$ ) les courbes de toutes les suites, pour le même $u_0$ fixé, et pour tous les réels q possibles.

2°) Déduisez-en leurs sens de variations

et leurs limites.

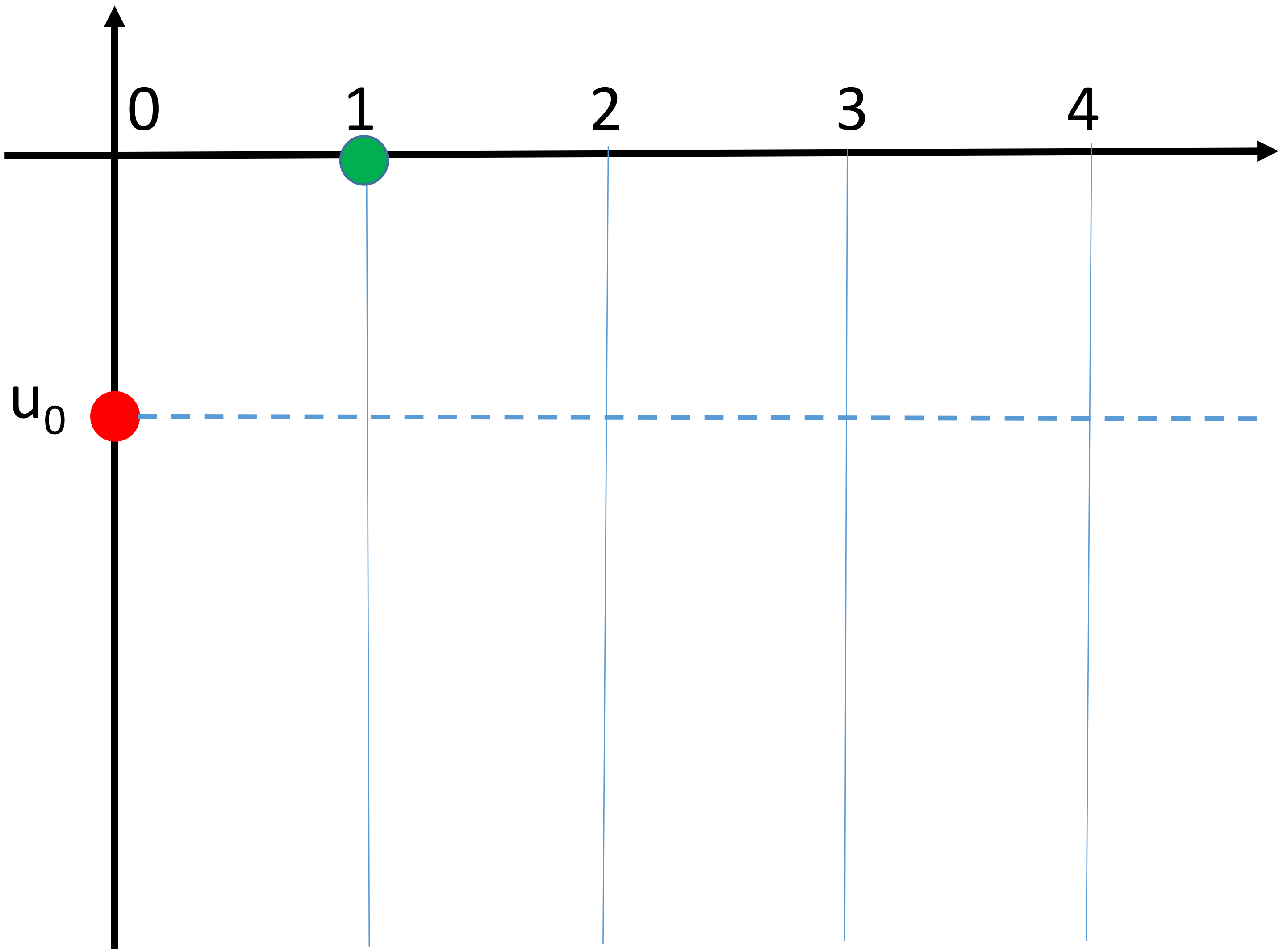
$u_{n+1} = u_n \times q$ $u_0 < 0$ 1[er] cas $q \geq 0$




$q = 0$

$u_0 < 0$

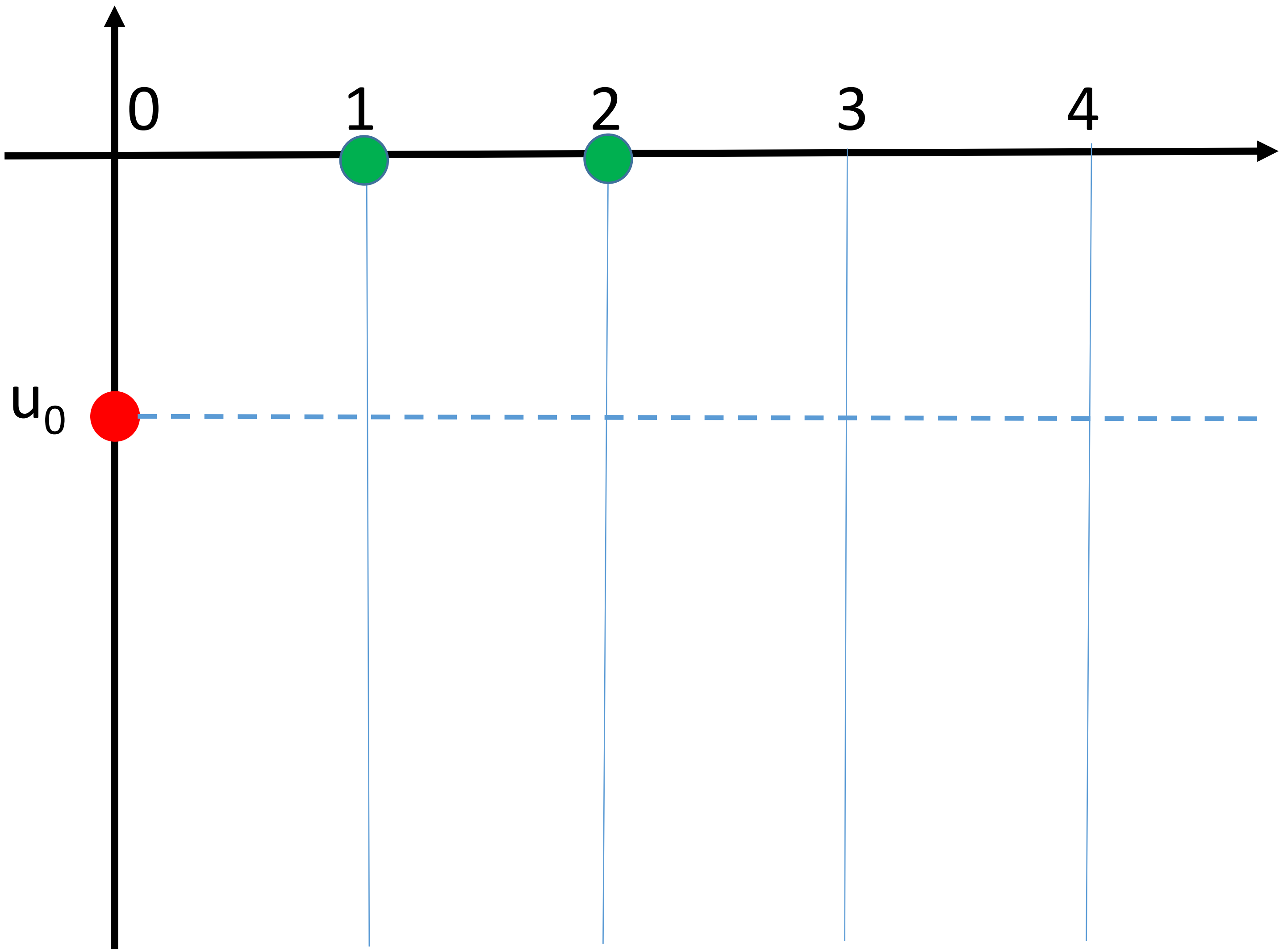
$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ 1$^{er}$ cas $q \geq 0$




$q = 0$

$u_1 = u_0 \times q = u_0 \times 0 = 0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ 1er cas $q \geq 0$

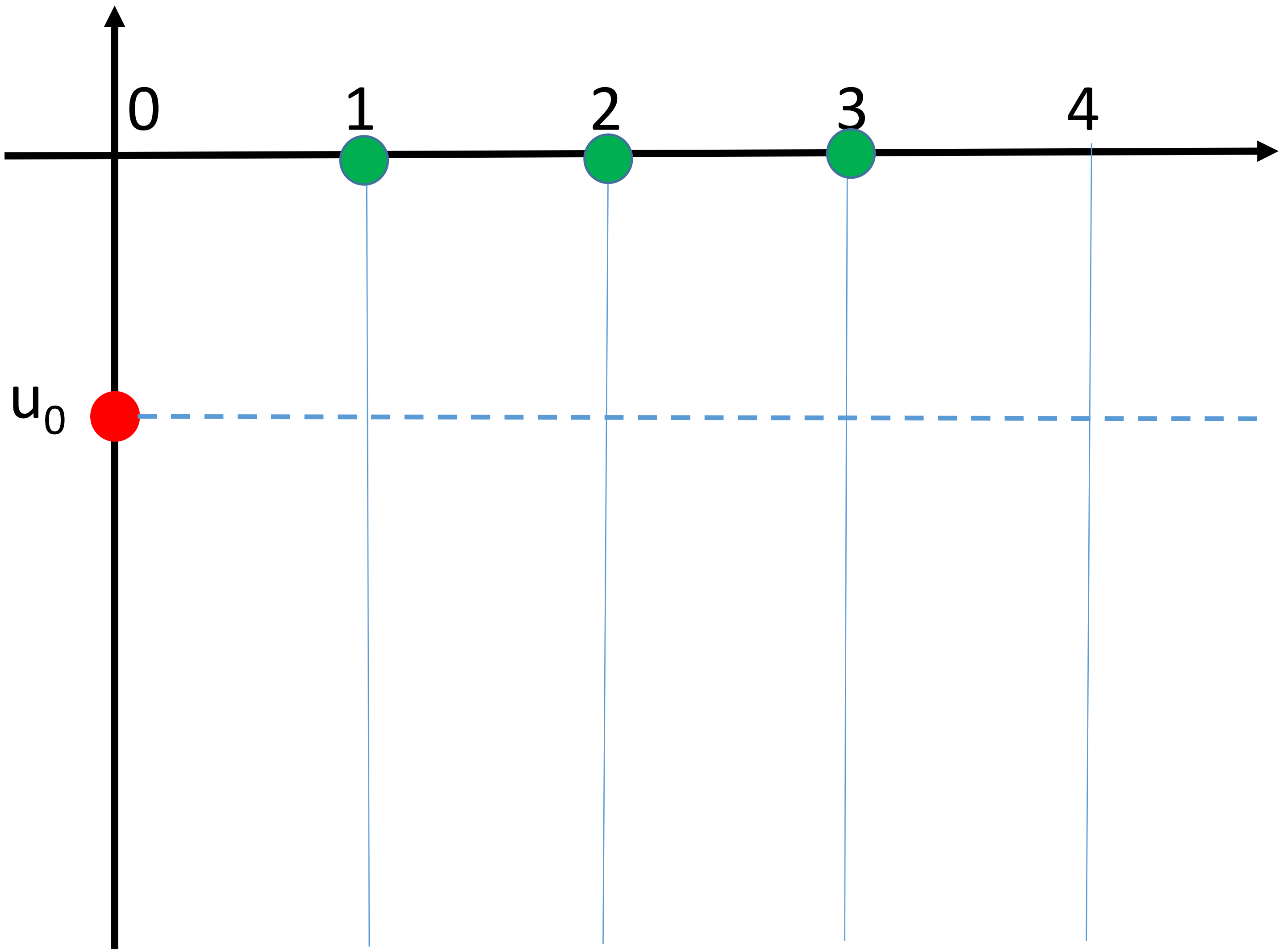



$q = 0$

$u_1 = u_0 \times q = u_0 \times 0 = 0$

$u_2 = u_1 \times q = 0 \times 0 = 0$

$u_{n+1} = u_n \times q$ $\qquad$ $u_0 < 0$ $\qquad$ $1^{er}$ cas $q \geq 0$



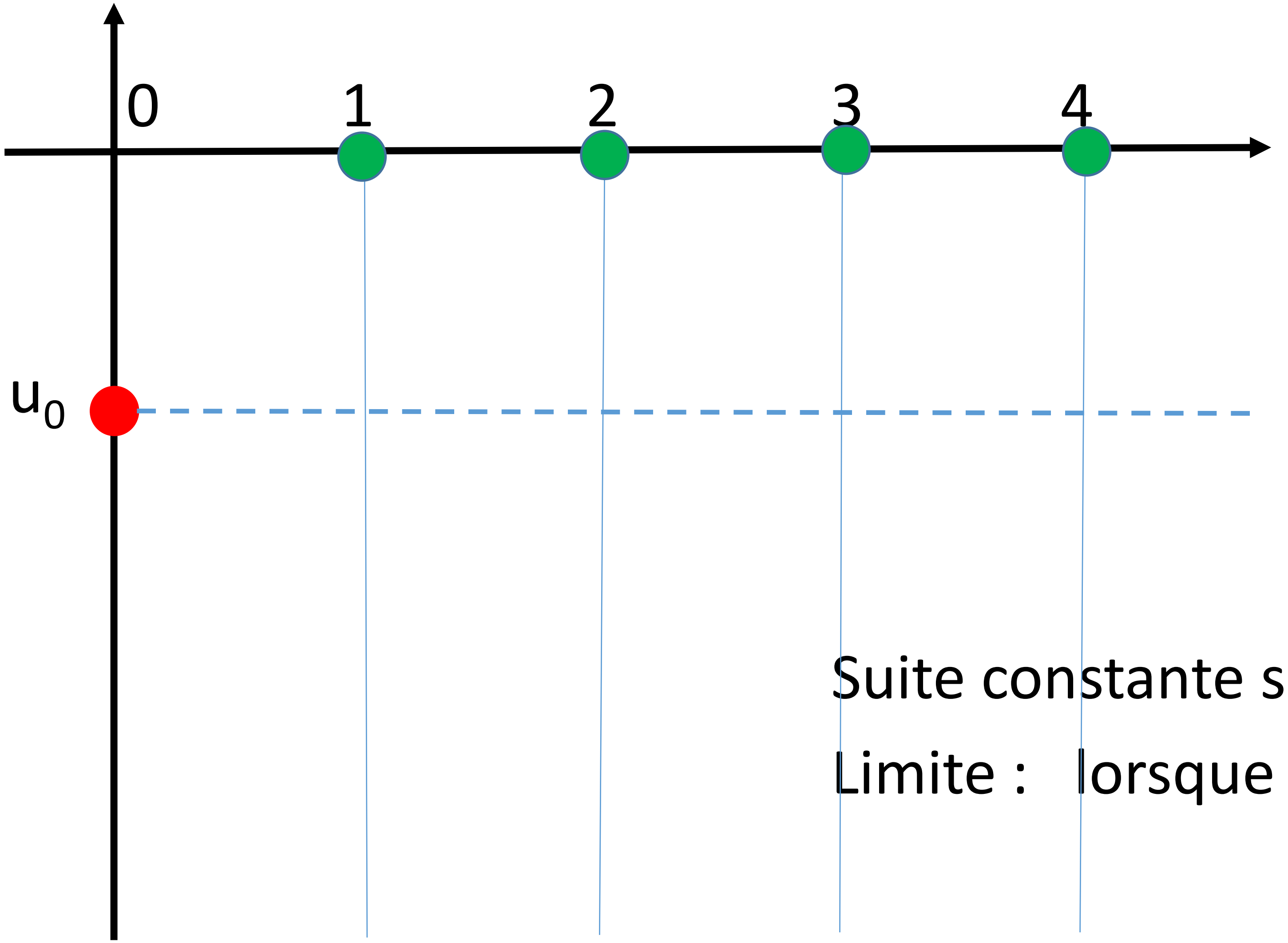

$q = 0$

$u_1 = u_0 \times q = u_0 \times 0 = 0$

$u_2 = u_1 \times q = 0 \times 0 = 0$

$u_3 = u_2 \times q = 0 \times 0 = 0$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ $1^{er}$ cas $q \geq 0$




$q = 0$

$u_1 = u_0 \times q = u_0 \times 0 = 0$

$u_2 = u_1 \times q = 0 \times 0 = 0$

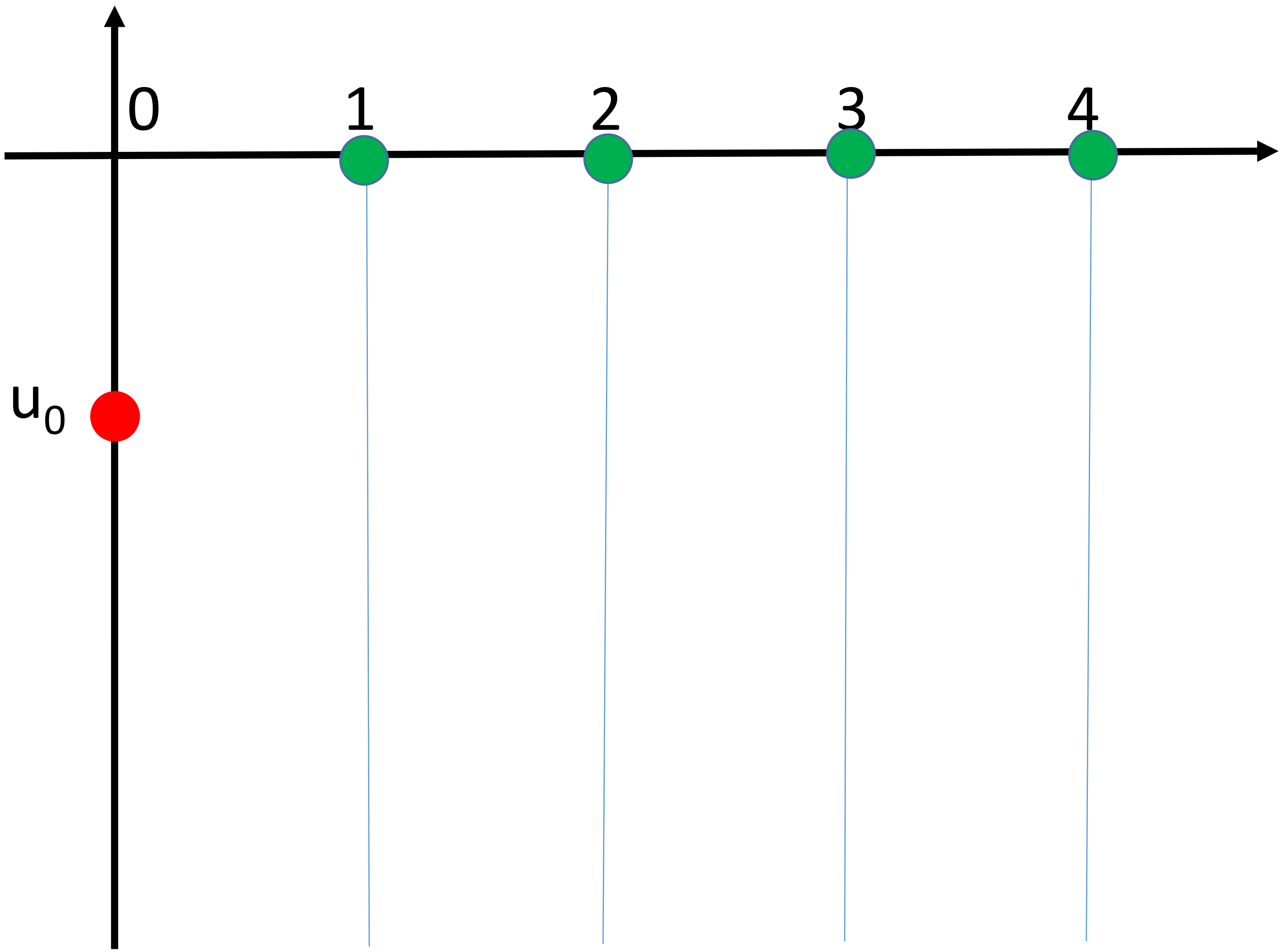
$u_3 = u_2 \times q = 0 \times 0 = 0$

$u_4 = u_3 \times q = 0 \times 0 = 0$

Suite constante sur $\mathbb{N}^*$

Limite : lorsque $n \to +\infty$ $\quad u_n \to 0$

$u_{n+1} = u_n \times q$ $\qquad u_0 < 0$ $\qquad 1^{er}$ cas $q \geq 0$

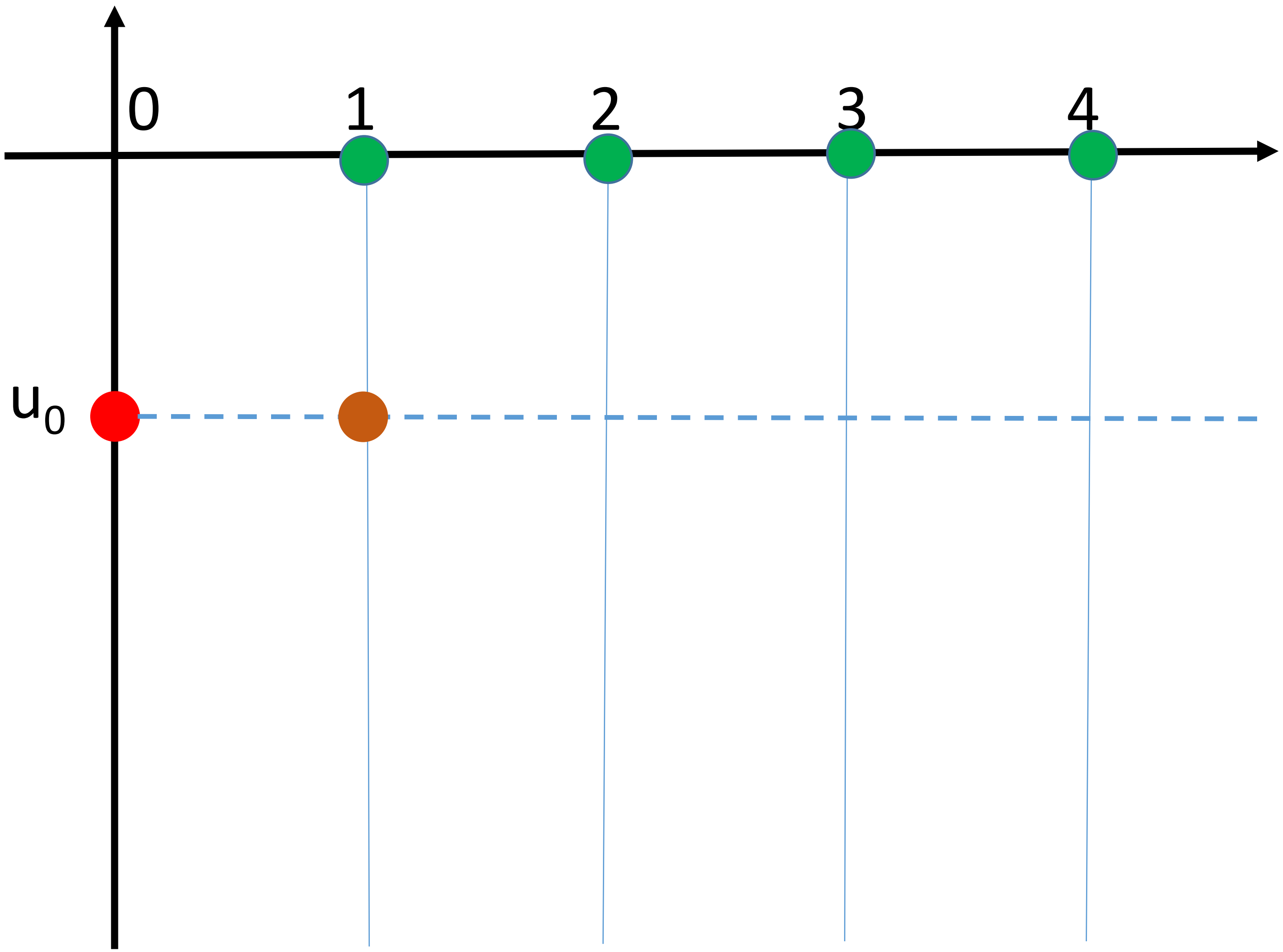



$q = 0$

$2^{ème}$ cas : $q = 1$

$u_0 < 0$

$u_{n+1} = u_n \times q$ $\quad$ $u_0 < 0$ $\quad$ 1^er^ cas $q \geq 0$

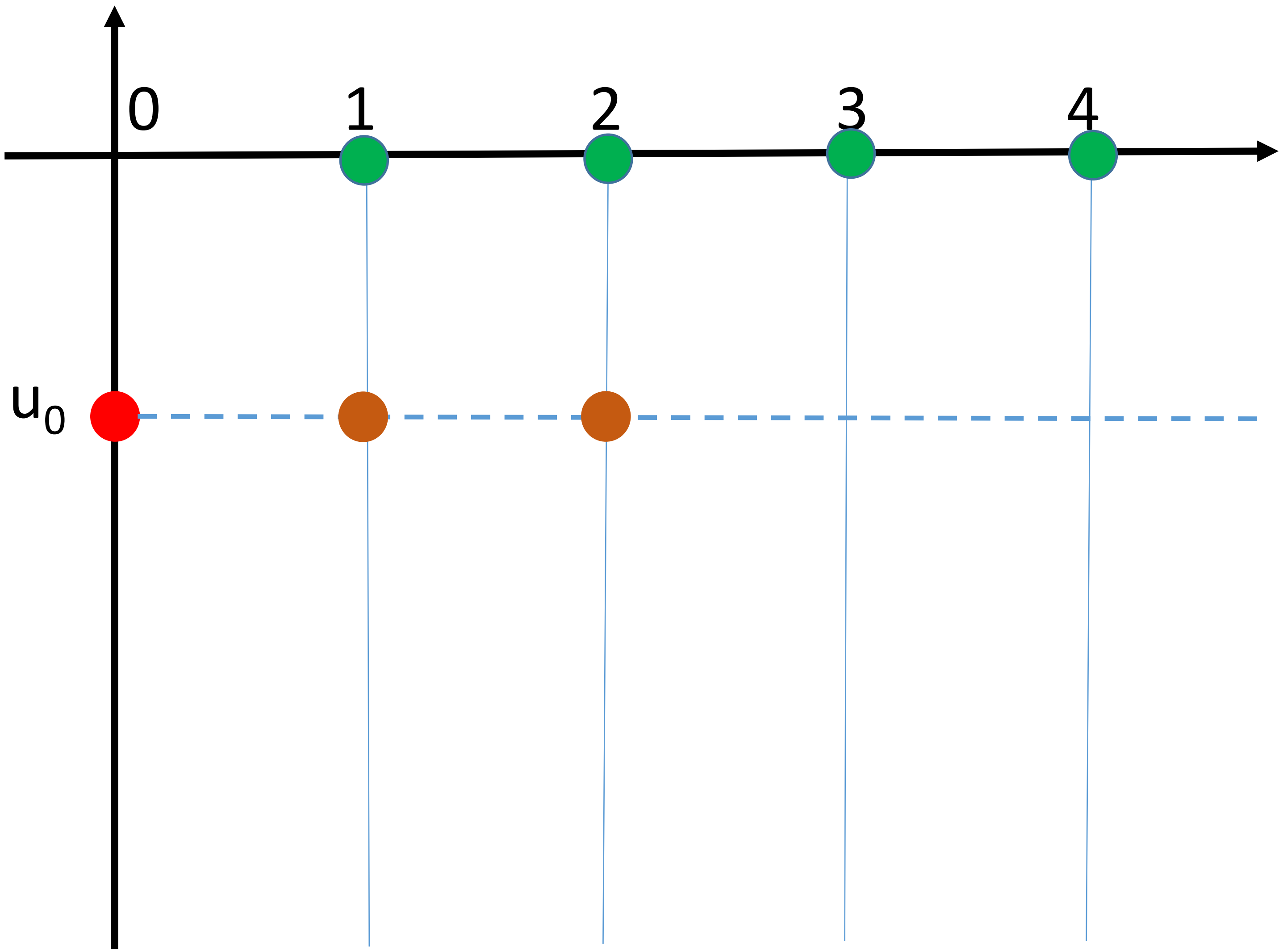



$q = 0$

$q = 1$

$u_1 = u_0 \times q = u_0 \times 1 = u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $1^{er}$ cas $q \geq 0$



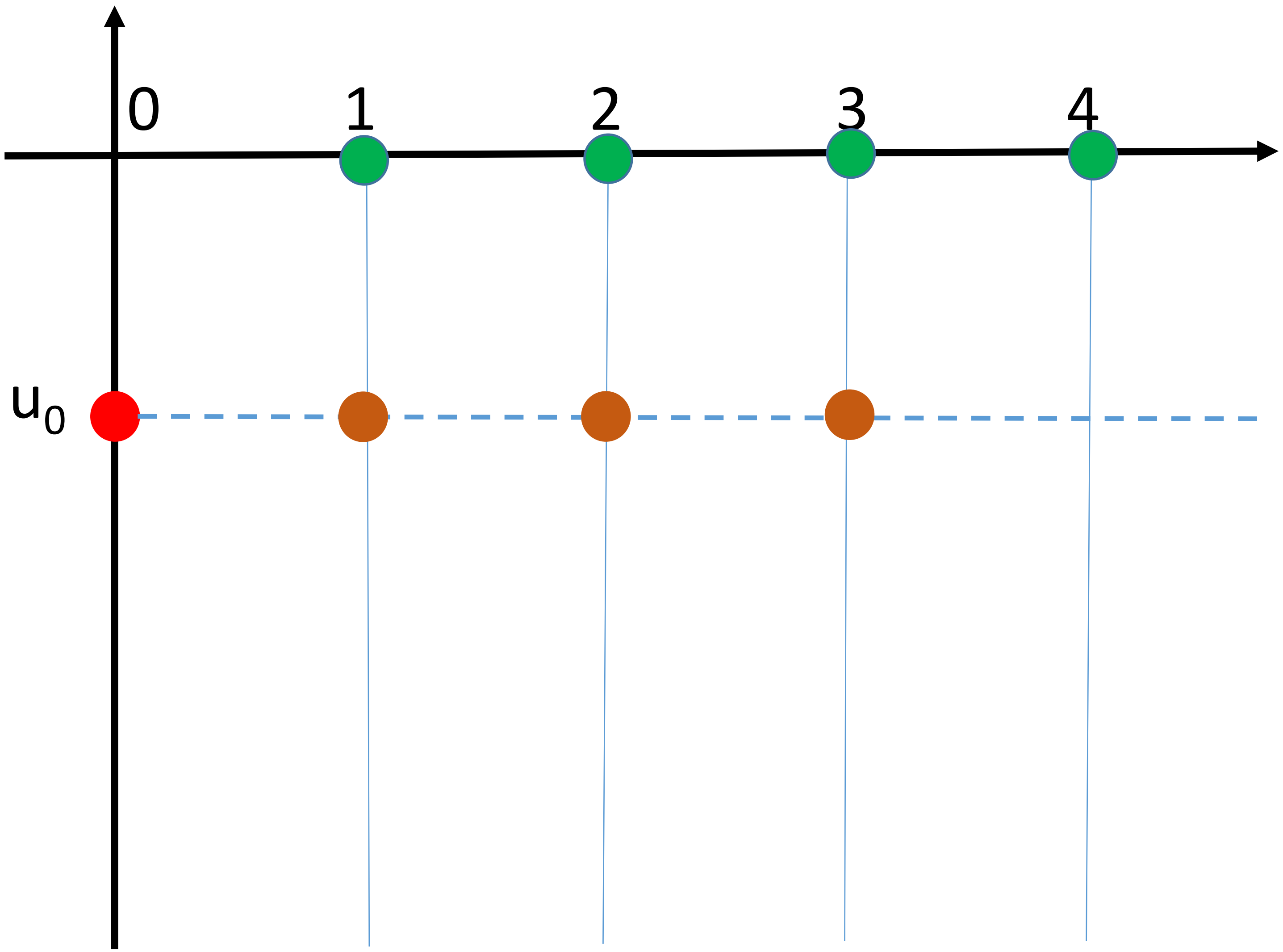

$q = 0$

$q = 1$

$u_1 = u_0 \times q = u_0 \times 1 = u_0$

$u_2 = u_1 \times q = u_0 \times 1 = u_0$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ $1^{er}$ cas $q \geq 0$




$q = 0$

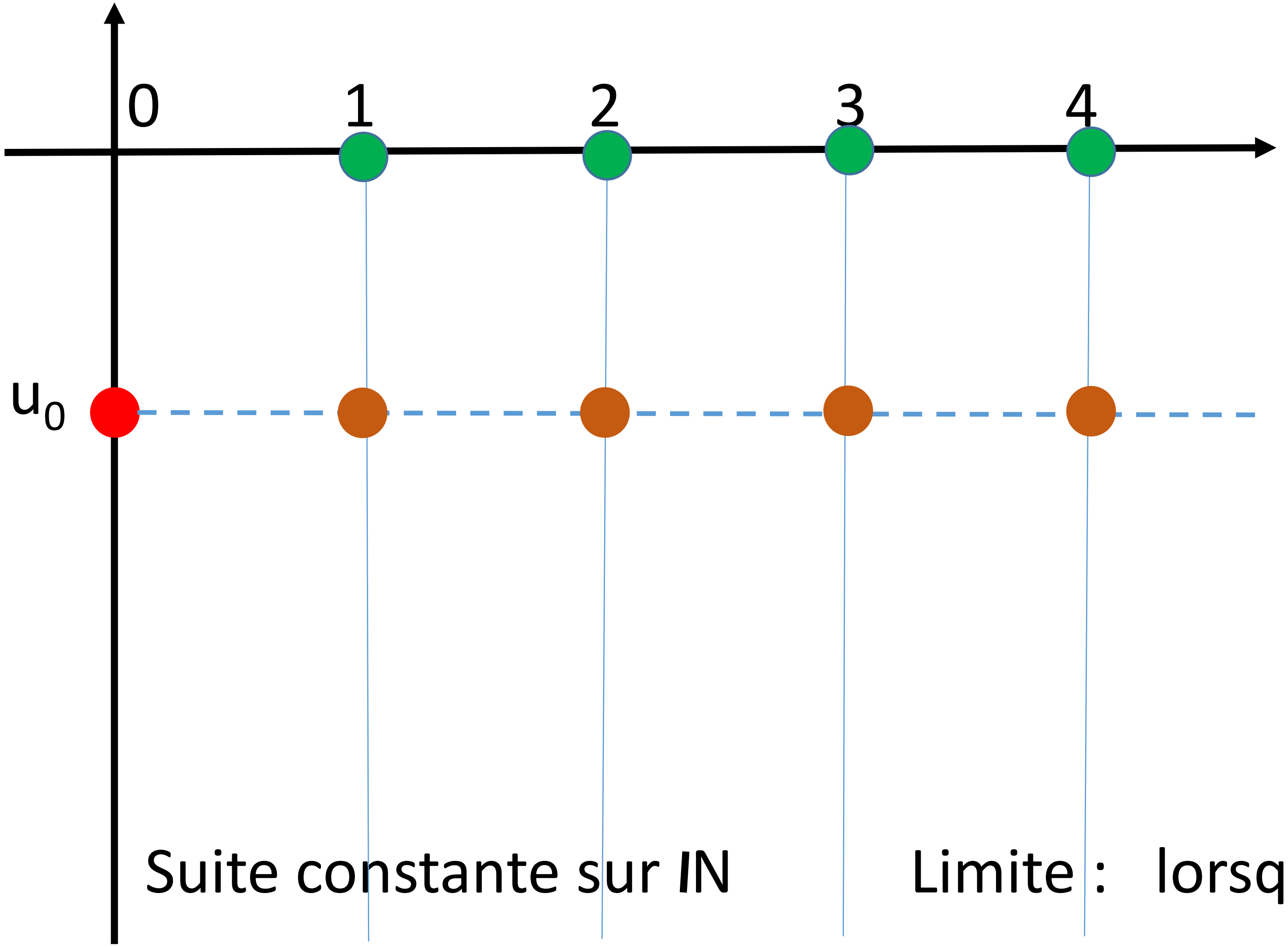
$q = 1$

$u_1 = u_0 \times q = u_0 \times 1 = u_0$

$u_2 = u_1 \times q = u_0 \times 1 = u_0$

$u_3 = u_2 \times q = u_0 \times 1 = u_0$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ 1er cas $q \geq 0$




$q = 0$

$q = 1$

$u_1 = u_0 \times q = u_0 \times 1 = u_0$

$u_2 = u_1 \times q = u_0 \times 1 = u_0$

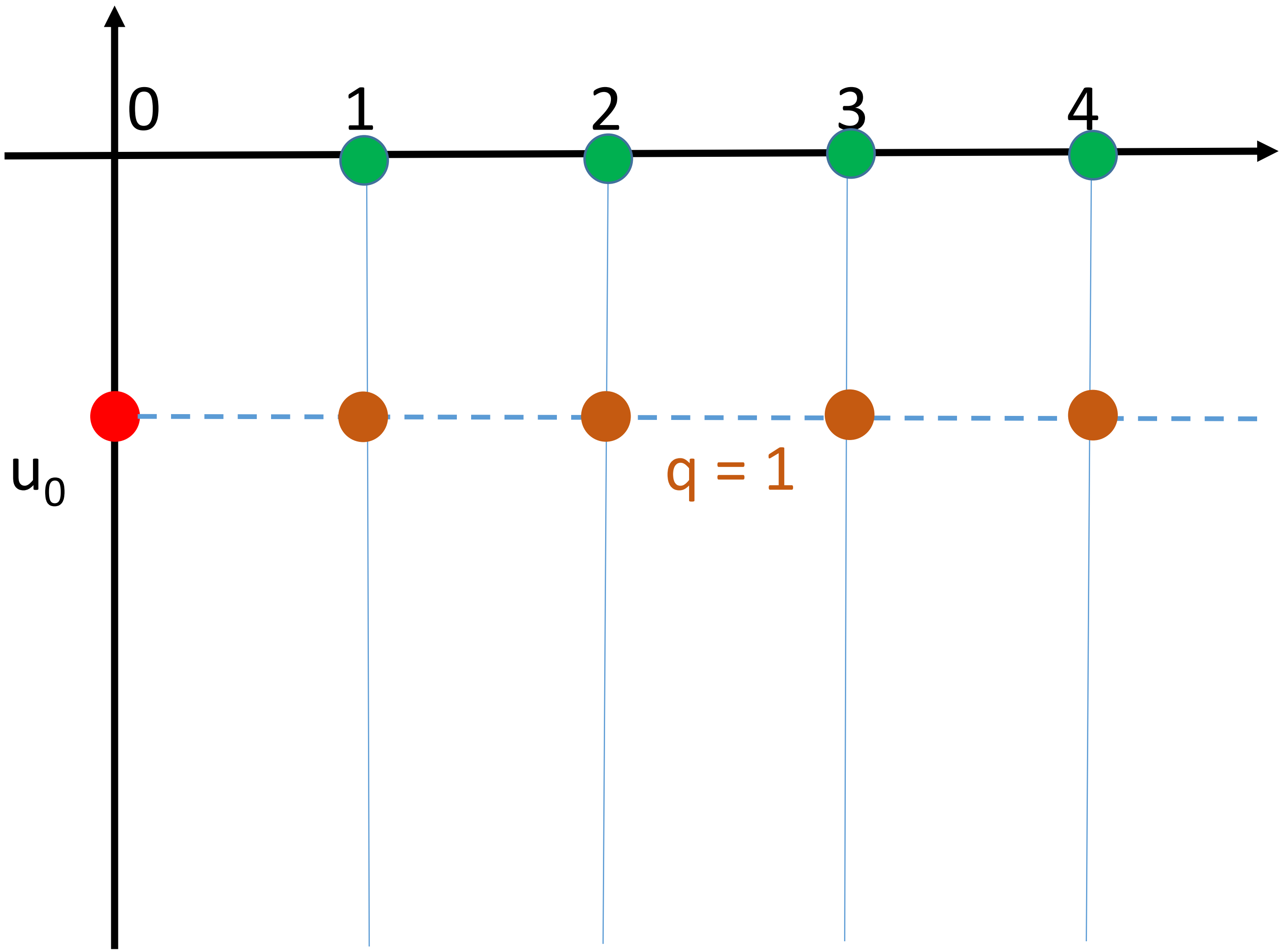
$u_3 = u_2 \times q = u_0 \times 1 = u_0$

$u_4 = u_3 \times q = u_0 \times 1 = u_0$

Suite constante sur *I*N

Limite : lorsque $n \to +\infty \quad u_n \to u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $1^{er}$ cas $q \geq 0$



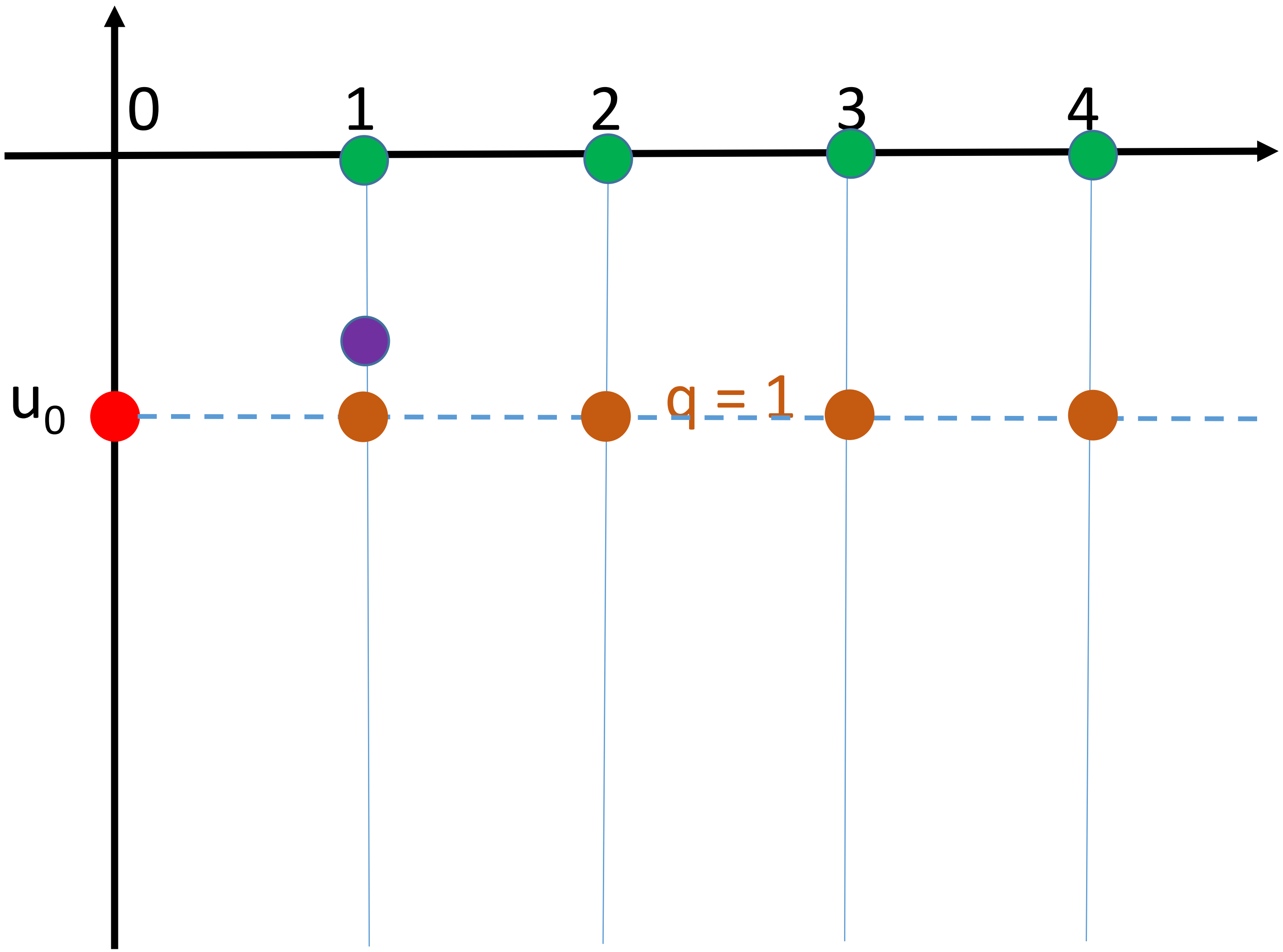

$q = 0$

$3^{ème}$ cas : $0 < q < 1$

par exemple,

tracé pour $q = 0,8$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $1^{er}$ cas $q \geq 0$



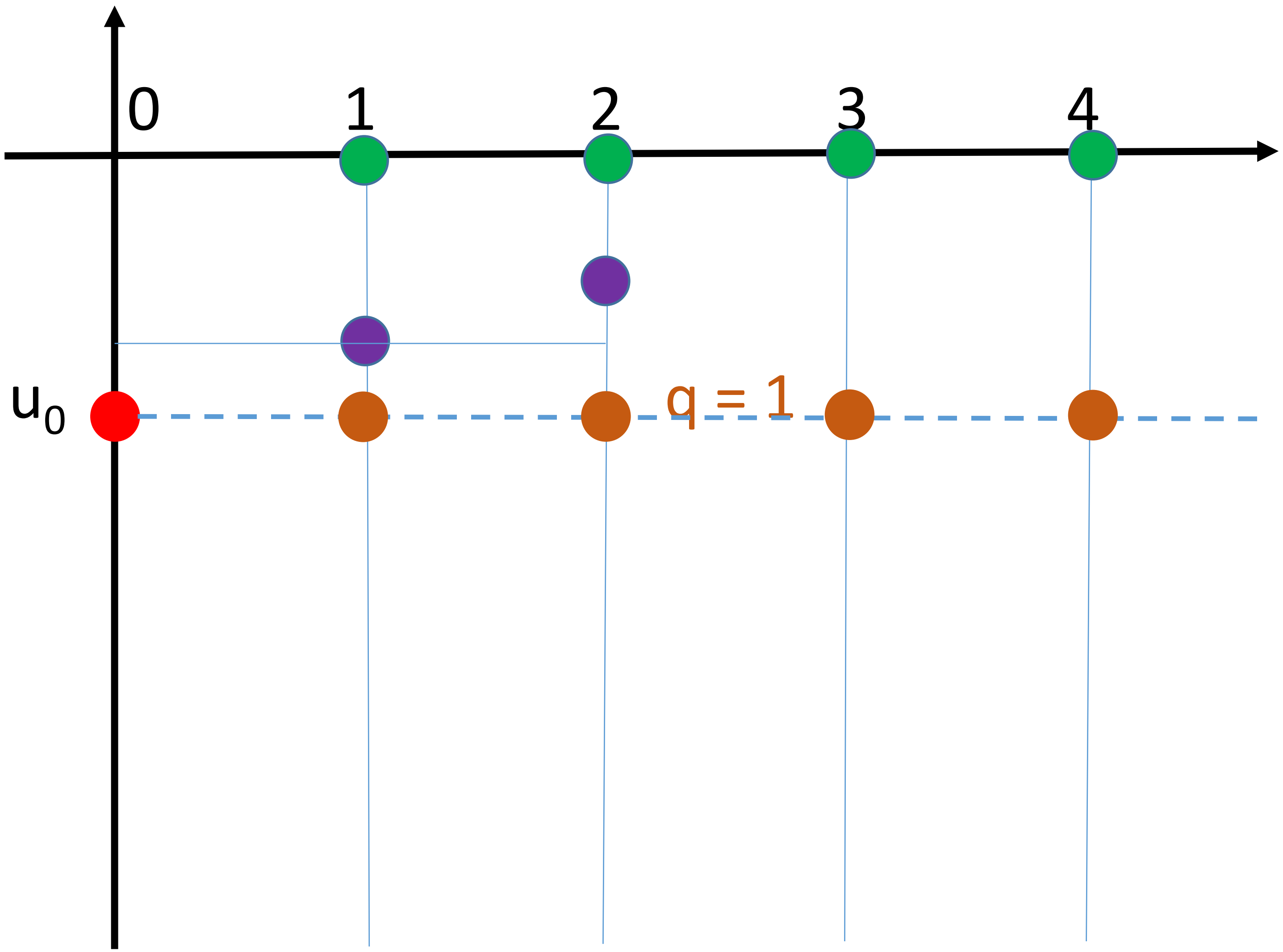

$3^{ème}$ cas : $q = 0$

$0 < q < 1$

$0 \times u_0 > q \times u_0 > 1 \times u_0$

$0 > u_1 > u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $1^{er}$ cas $q \geq 0$




$q = 0$

$0 < q < 1$

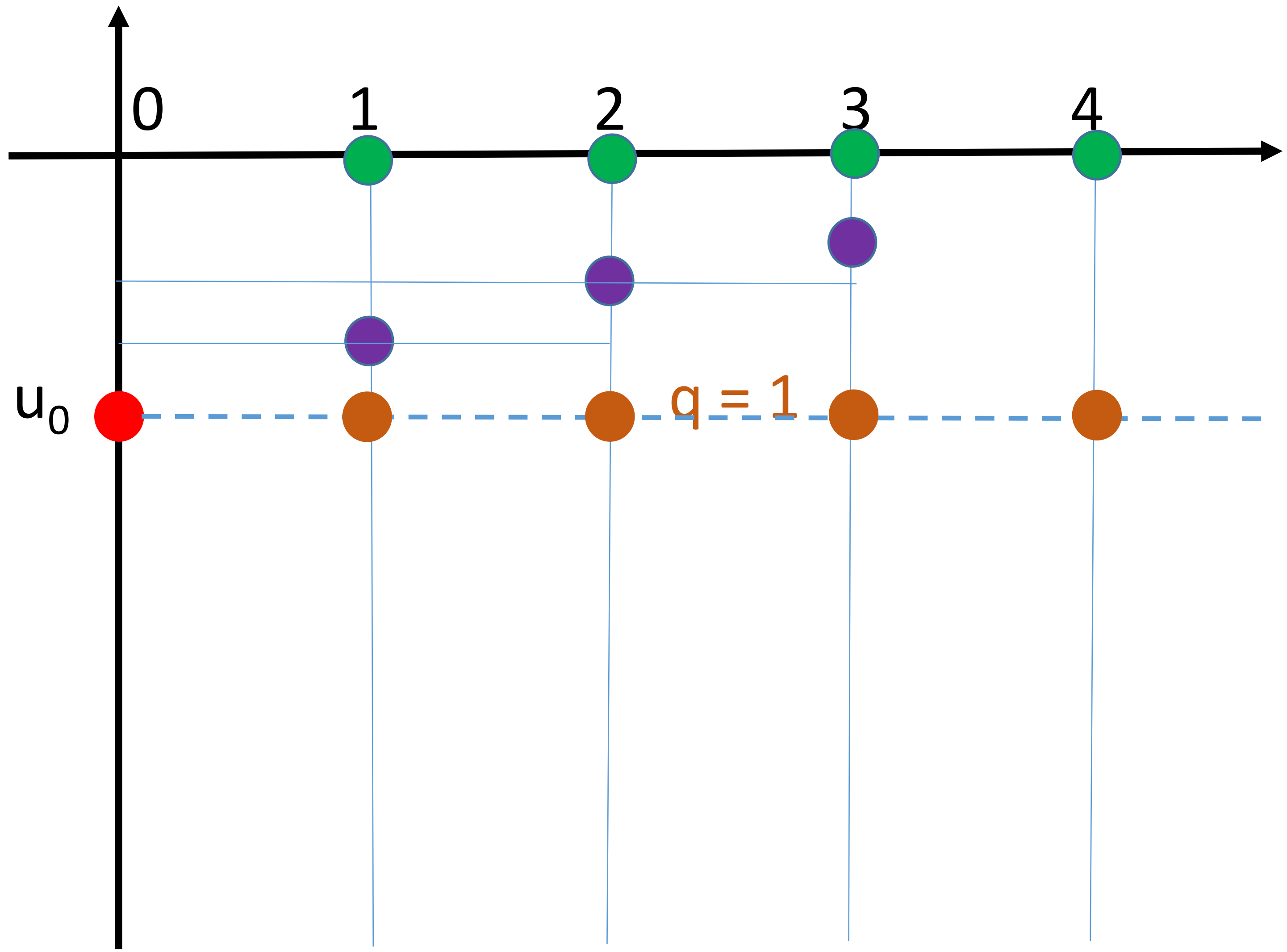
$0 \times u_0 > q \times u_0 > 1 \times u_0$

$0 > u_1 > u_0$

$0 \times q > u_1 \times q > u_0 \times q$

$0 > u_2 > u_1$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ 1$^{er}$ cas $q \geq 0$




$q = 0$

$0 < q < 1$

$0 \times u_0 > q \times u_0 > 1 \times u_0$

$0 > u_1 > u_0$

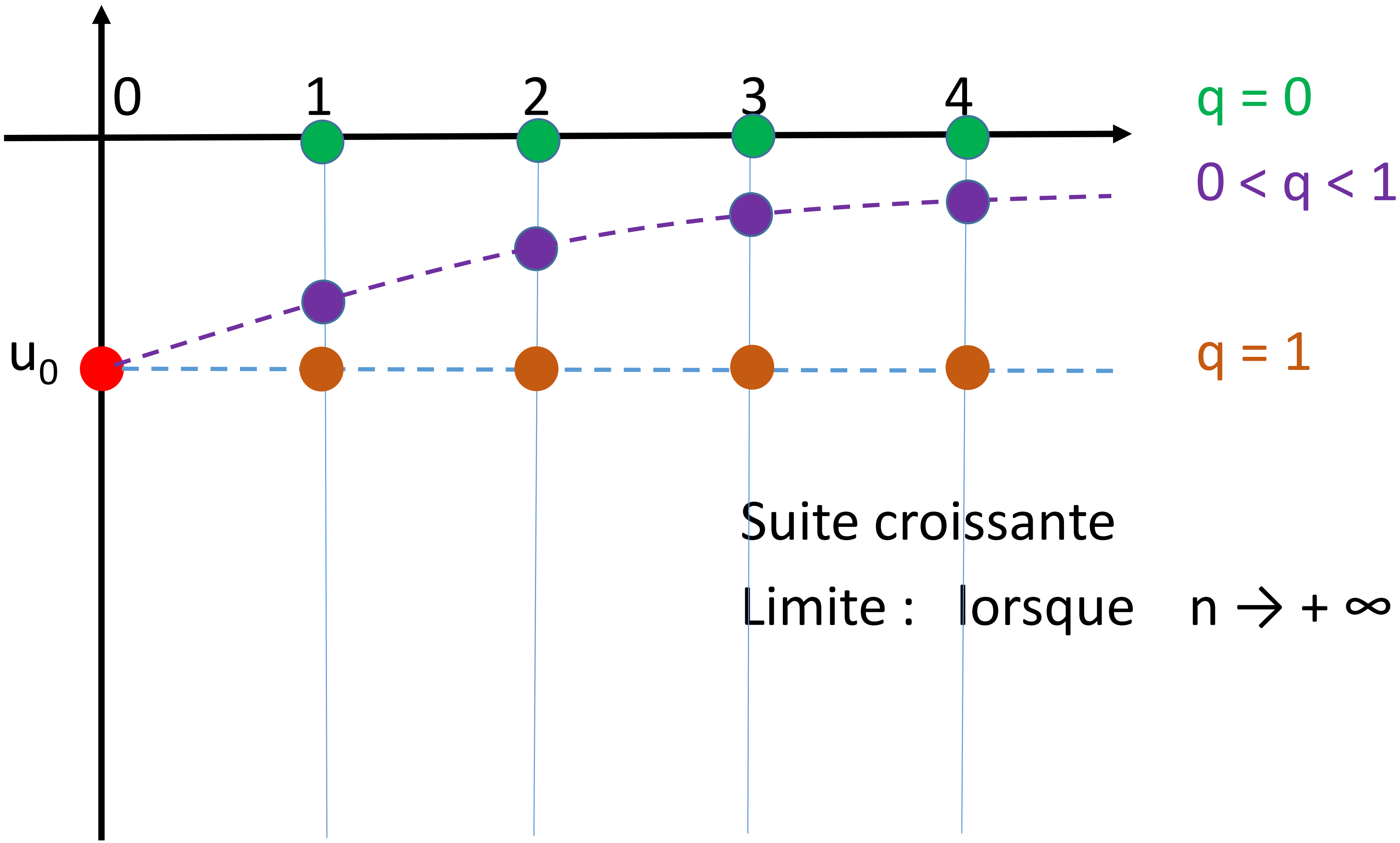
$0 \times q > u_1 \times q > u_0 \times q$

$0 > u_2 > u_1$

$0 \times q > u_2 \times q > u_1 \times q$

$0 > u_3 > u_2$

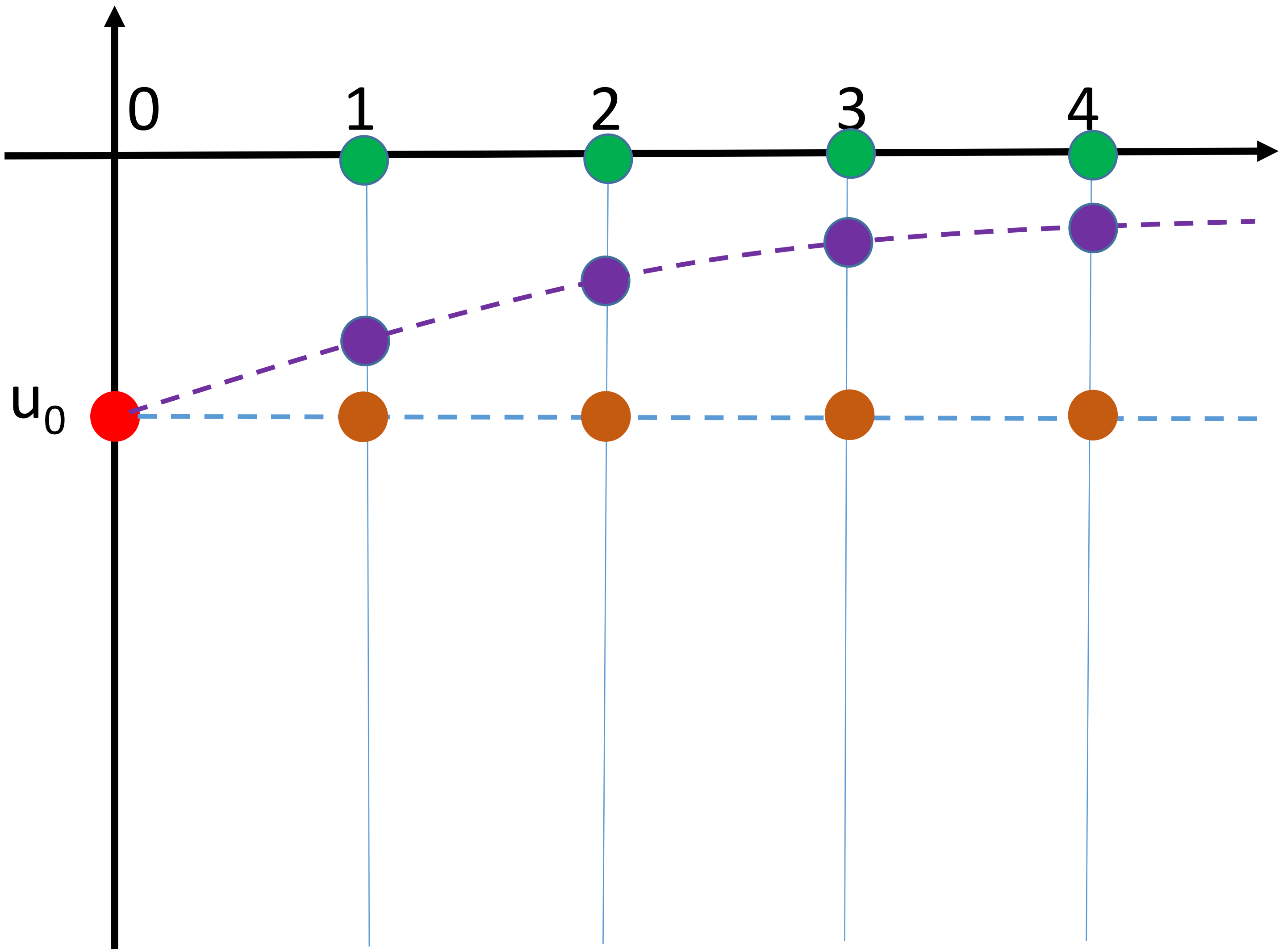
$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ $1^{er}$ cas $q \geq 0$




Suite croissante

Limite : lorsque $n \to +\infty \quad u_n \to 0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ 1er cas $q \geq 0$




$q = 0$

$0 < q < 1$

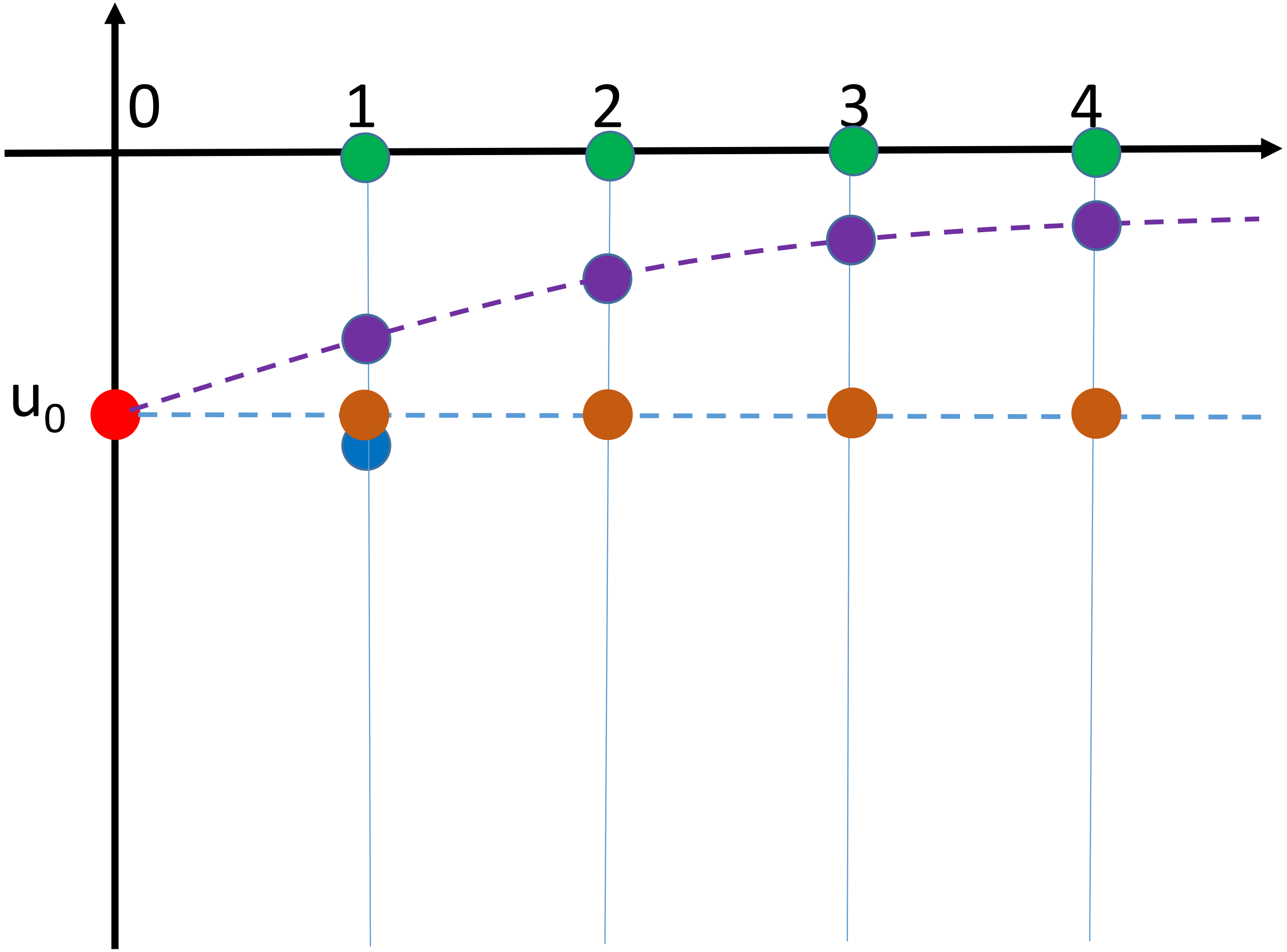
$q = 1$

4ème cas : $q > 1$

par exemple,

tracé pour $q = 1,2$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ $1^{er}$ cas $q \geq 0$




$q = 0$

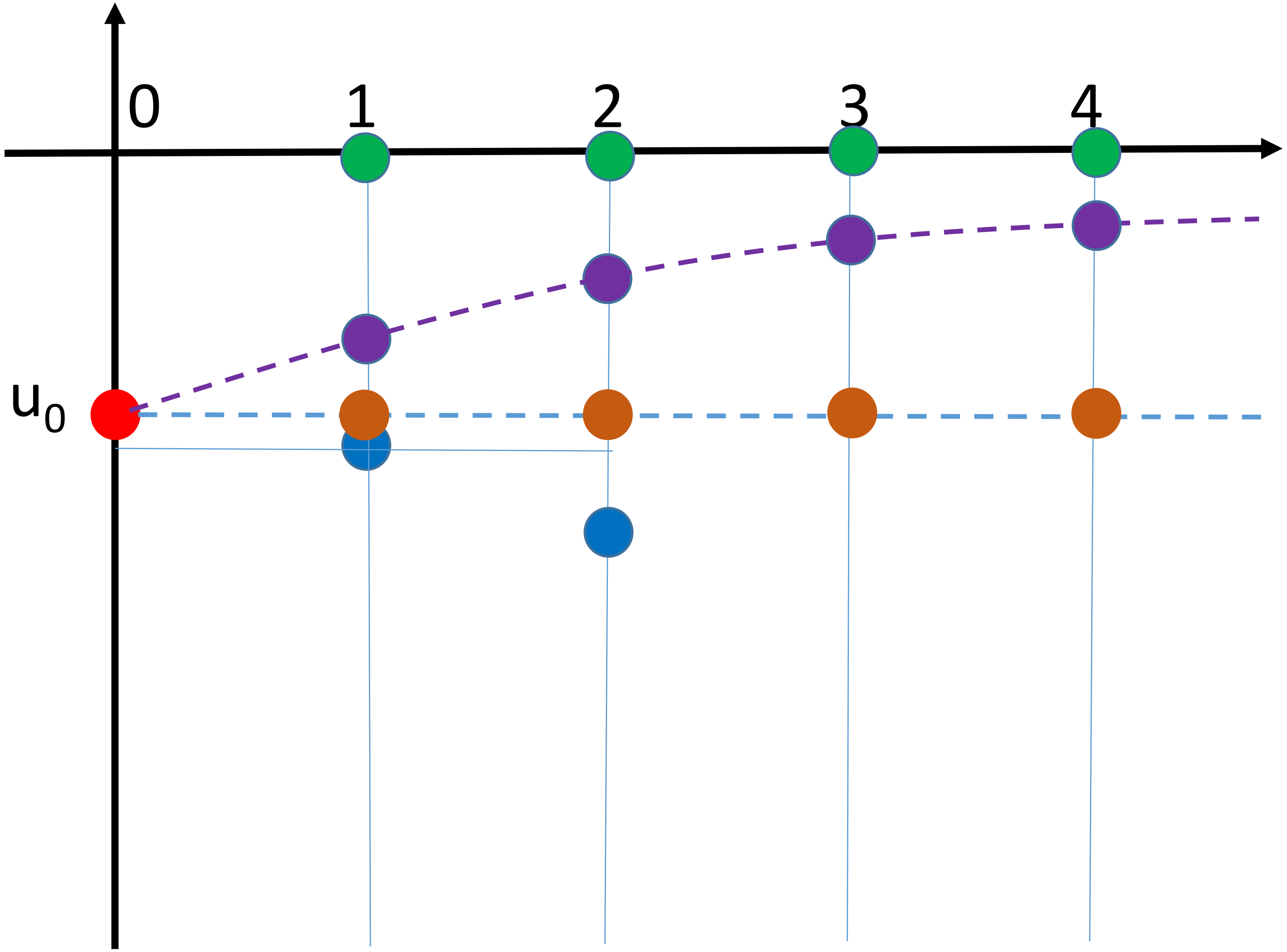
$0 < q < 1$

$q = 1$

$4^{ème}$ cas : $q > 1$

$q \times u_0 > 1 \times u_0$ $\quad u_1 < u_0$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ $1^{er}$ cas $q \geq 0$




$q = 0$

$0 < q < 1$

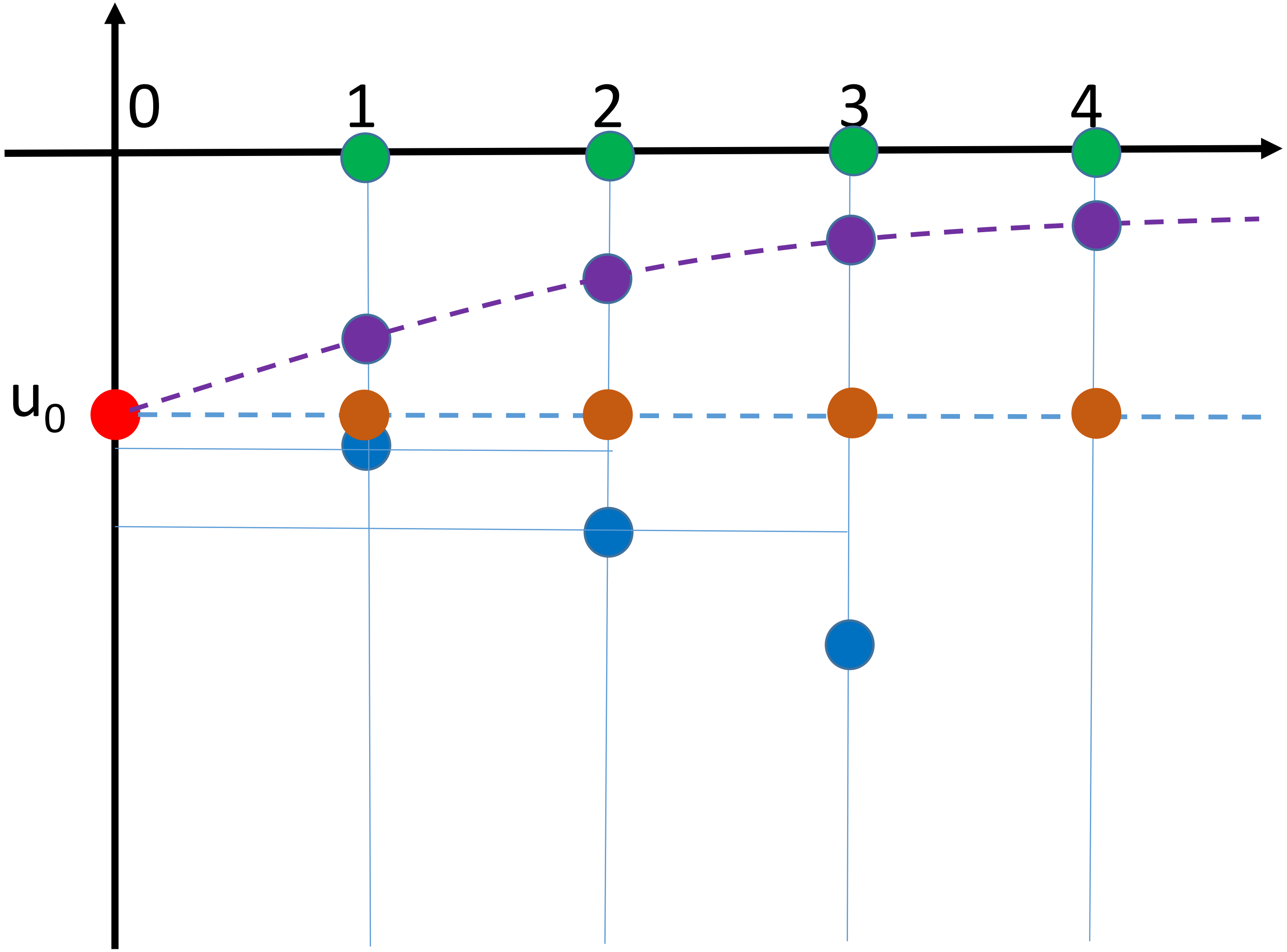
$q = 1$

$q > 1$

$q \times u_0 > 1 \times u_0 \qquad u_1 < u_0$

$q \times u_1 < q \times u_0 \qquad u_2 < u_1$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ 1er cas $q \geq 0$




$q = 0$

$0 < q < 1$

$q = 1$

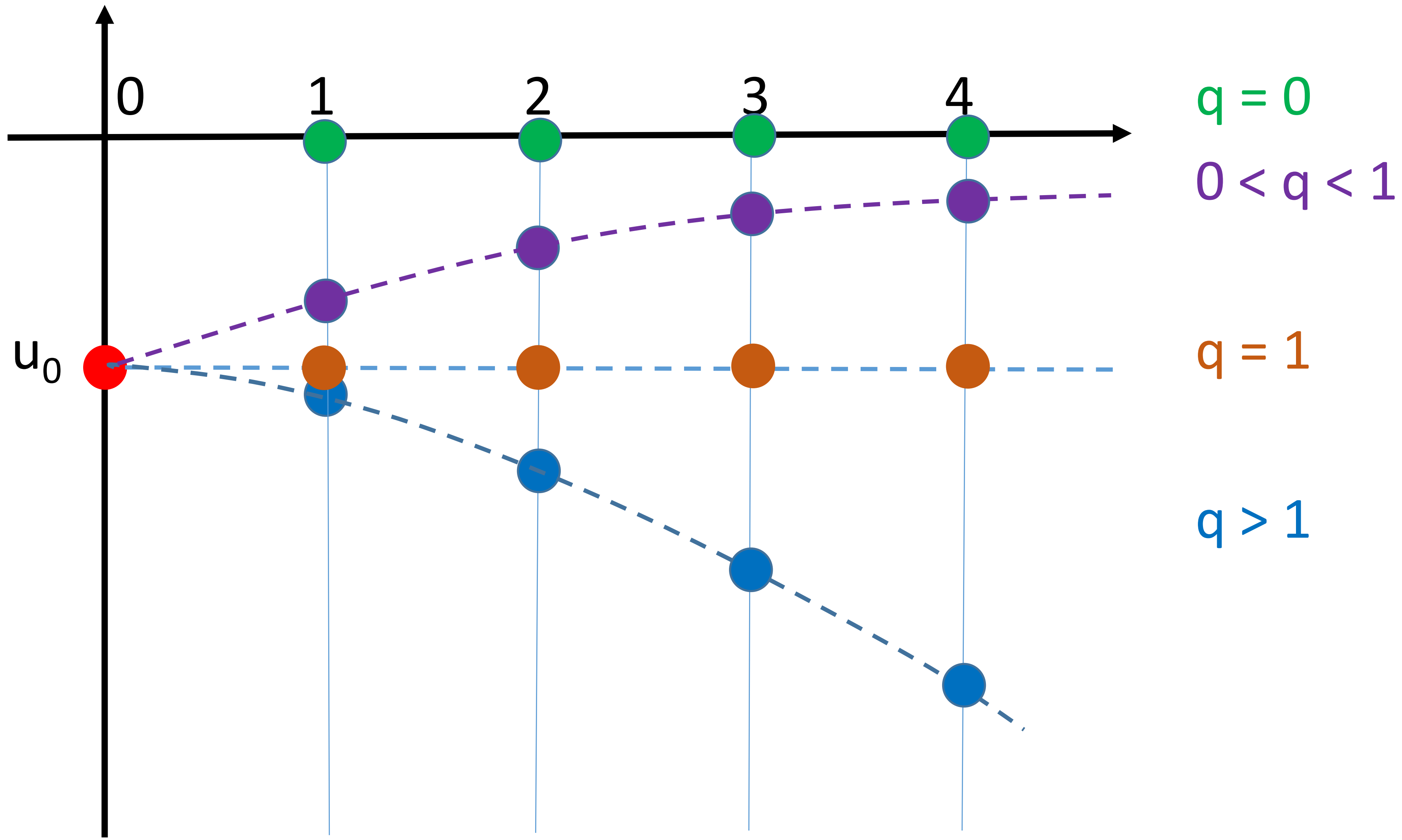
$q > 1$

$q \times u_0 > 1 \times u_0 \qquad u_1 < u_0$

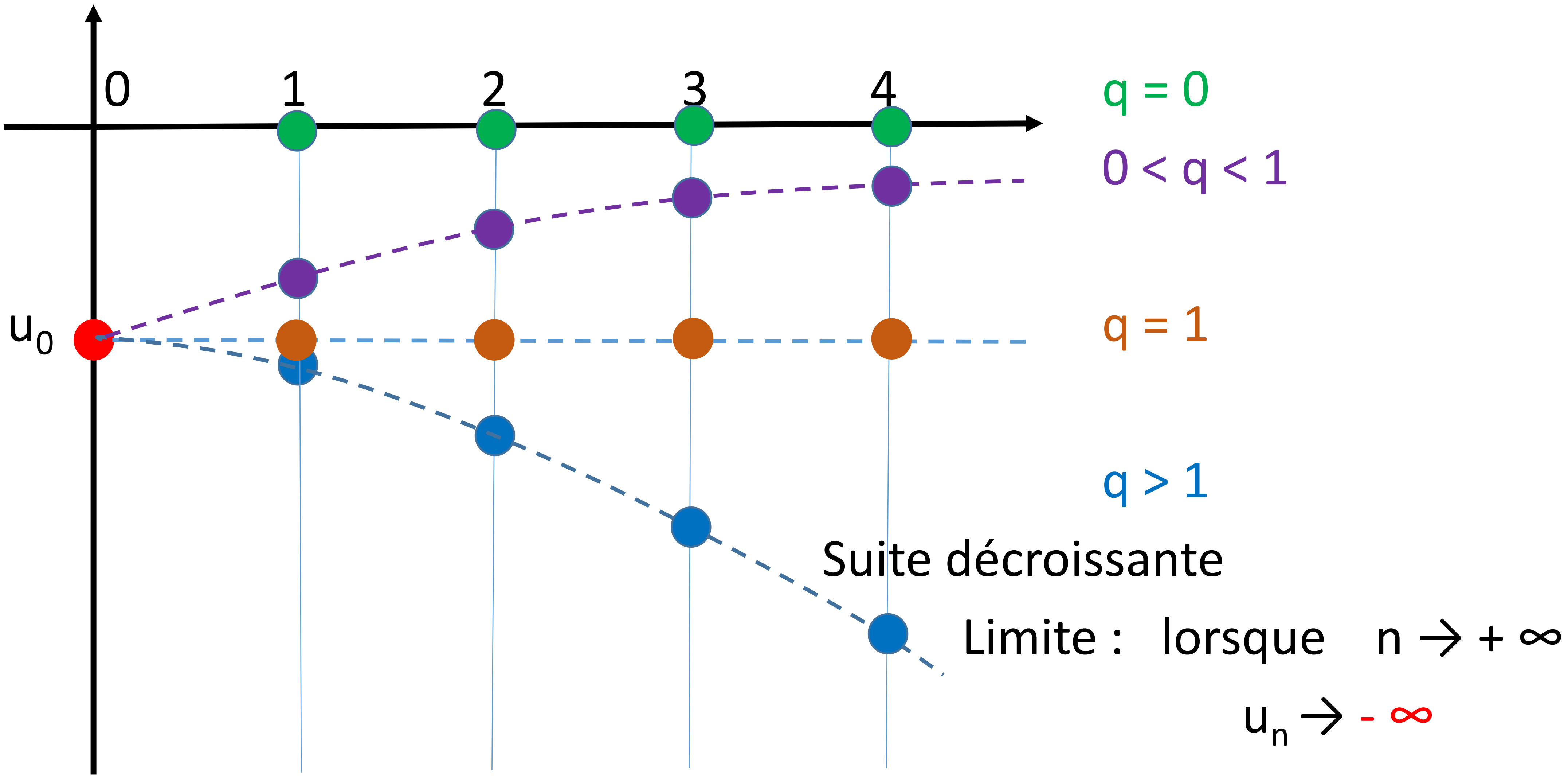
$q \times u_1 < q \times u_0 \qquad u_2 < u_1$

$q \times u_2 < q \times u_1 \qquad u_3 < u_2$

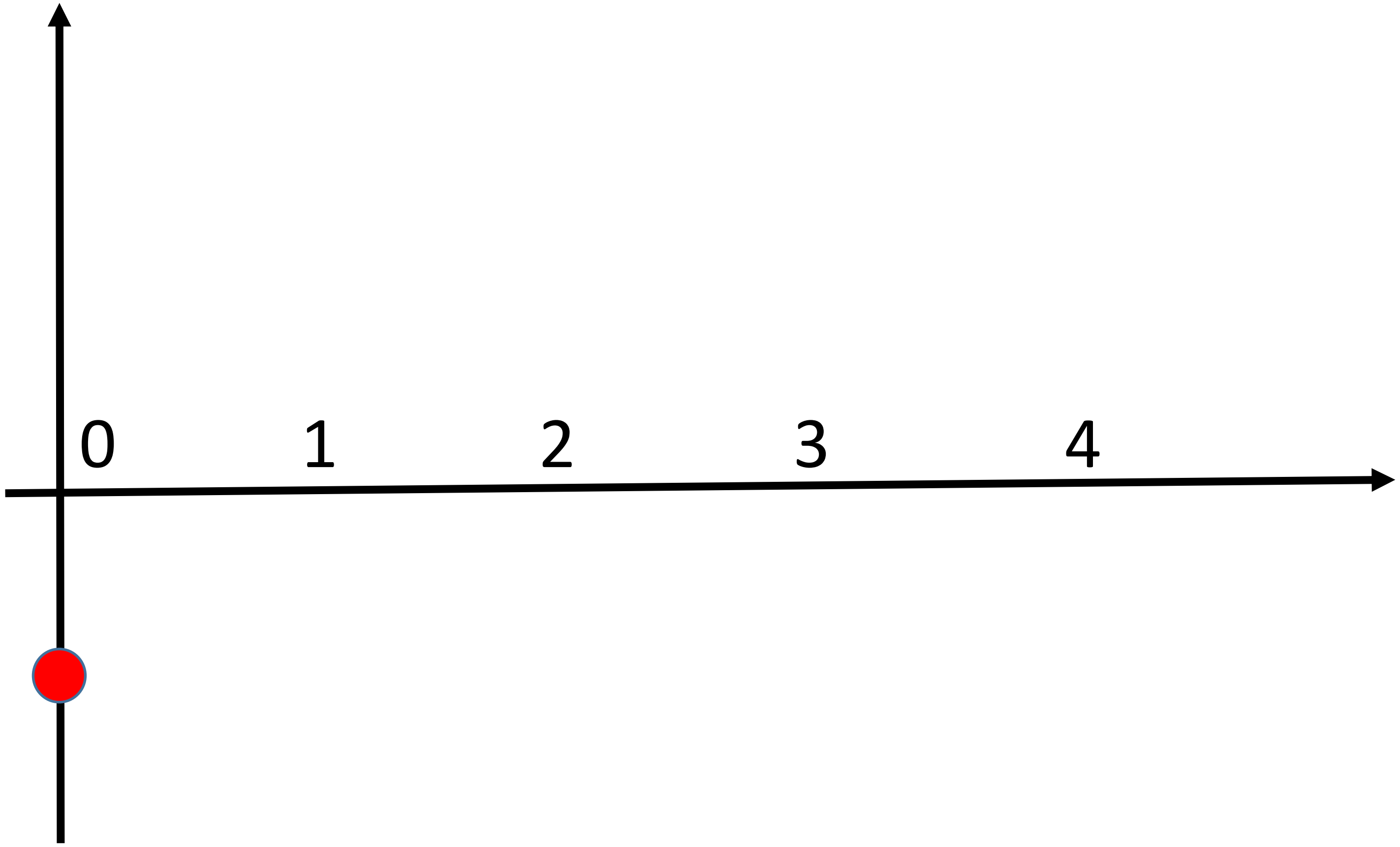
$u_{n+1} = u_n \times q$ $u_0 < 0$ 1^er^ cas $q \geq 0$



$u_{n+1} = u_n \times q$ $u_0 < 0$ $1^{er}$ cas $q \geq 0$



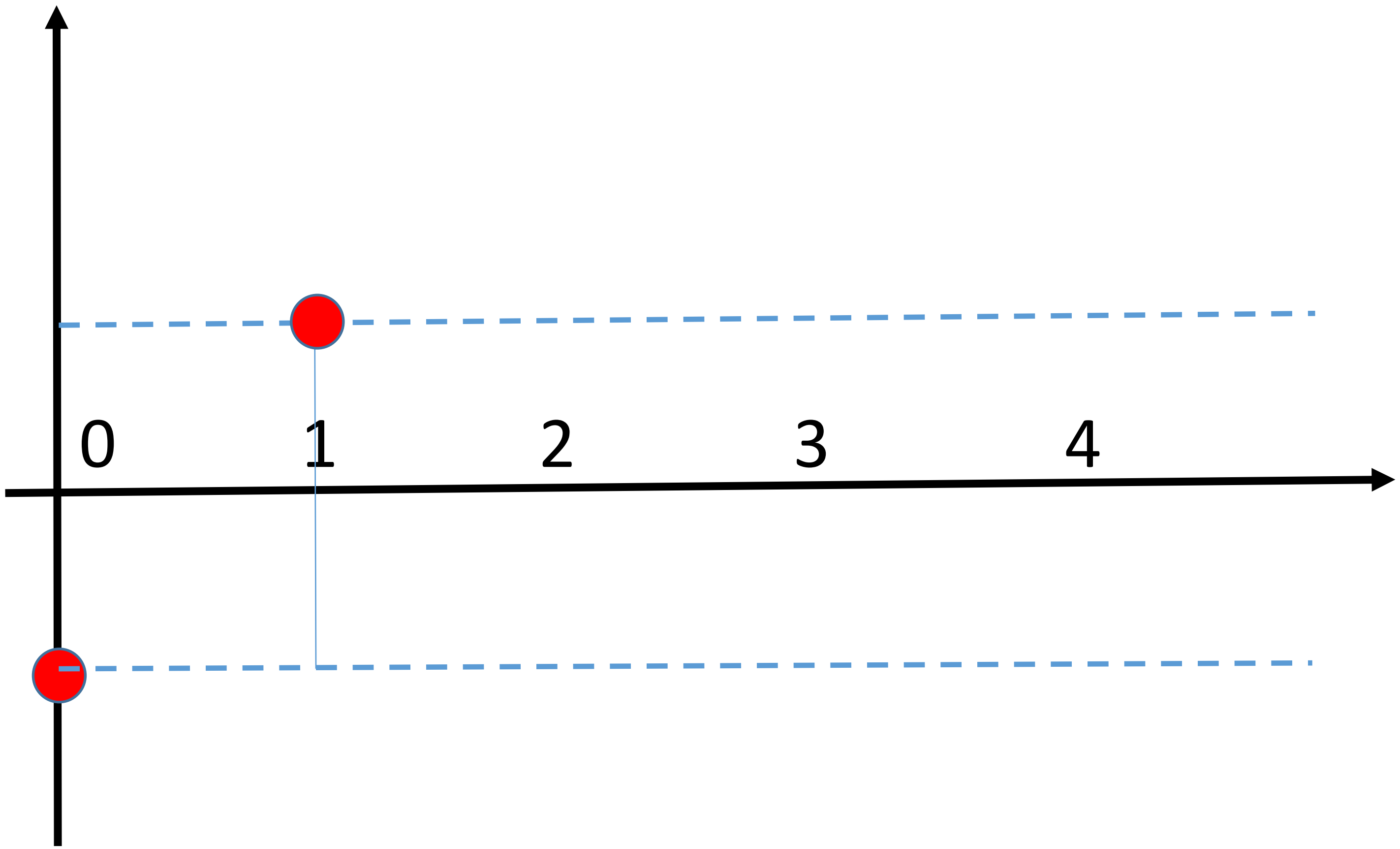
$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$




$5^{\text{ème}}$ cas : $q = -1$

$u_0 < 0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$

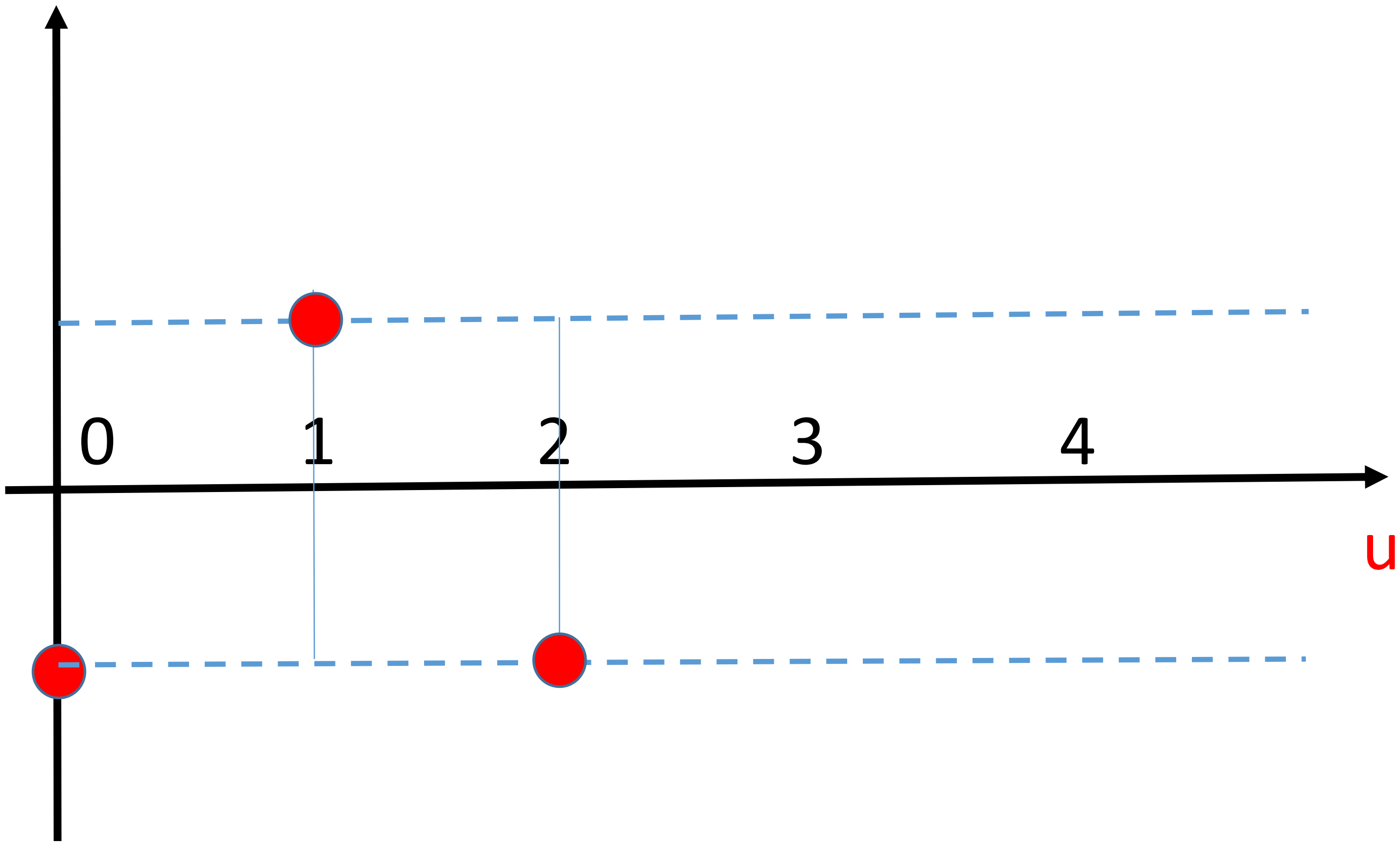



$q = -1$

$u_0 < 0$

$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$



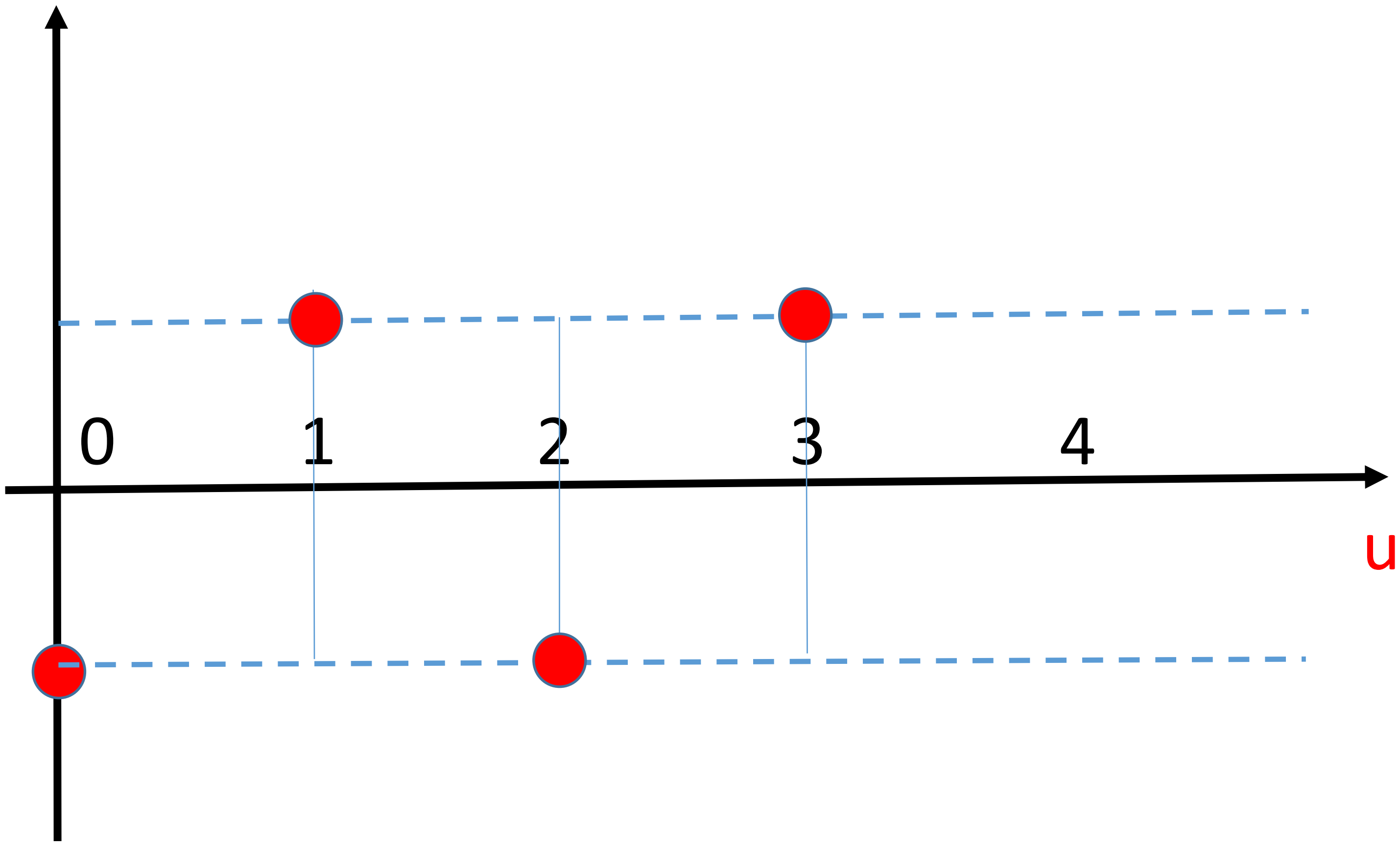

$q = -1$

$u_0 < 0$

$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$

$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$




$q = -1$

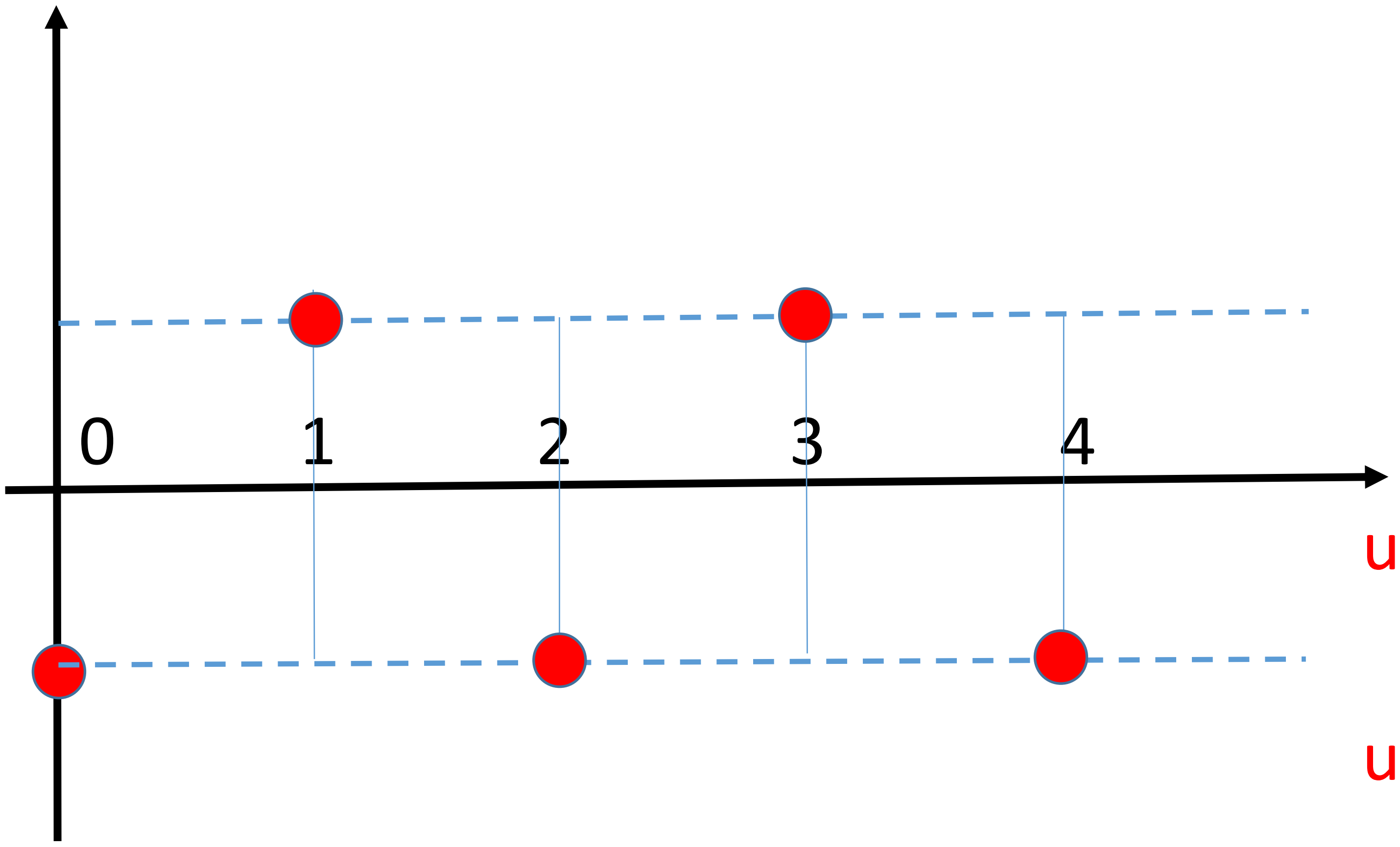
$u_0 < 0$

$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$

$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$

$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$




$q = -1$

$u_0 < 0$

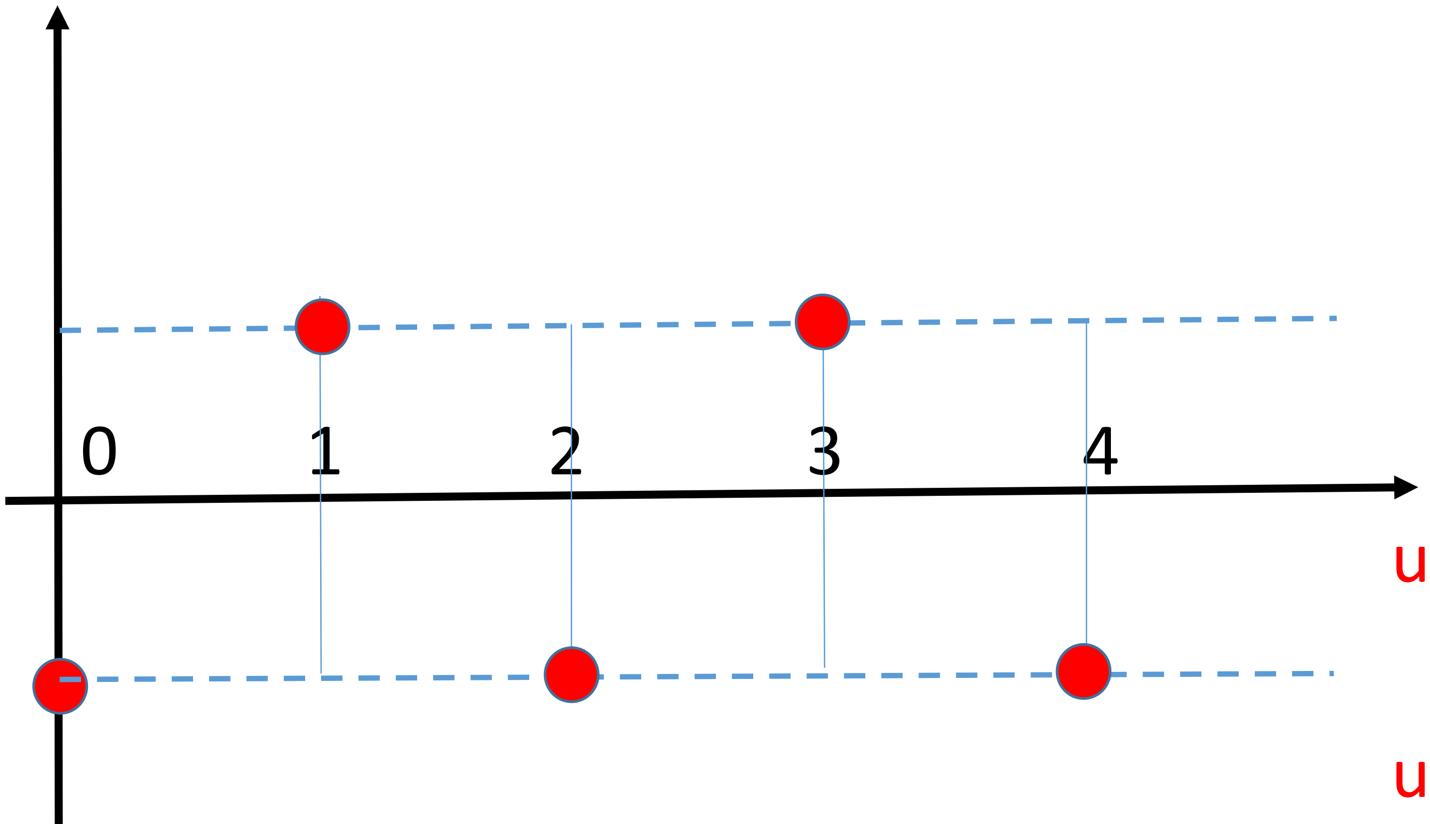
$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$

$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$

$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$

$u_4 = q \times u_3 = (-1) \times (-u_0) = u_0$

$u_{n+1} = u_n \times q$ $\quad$ $u_0 < 0$ $\quad$ $2^{\text{ème}}$ cas $q < 0$




$q = -1$

$u_0 < 0$

$u_1 = q \times u_0 = (-1) \times u_0 = -u_0$

$u_2 = q \times u_1 = (-1) \times (-u_0) = u_0$

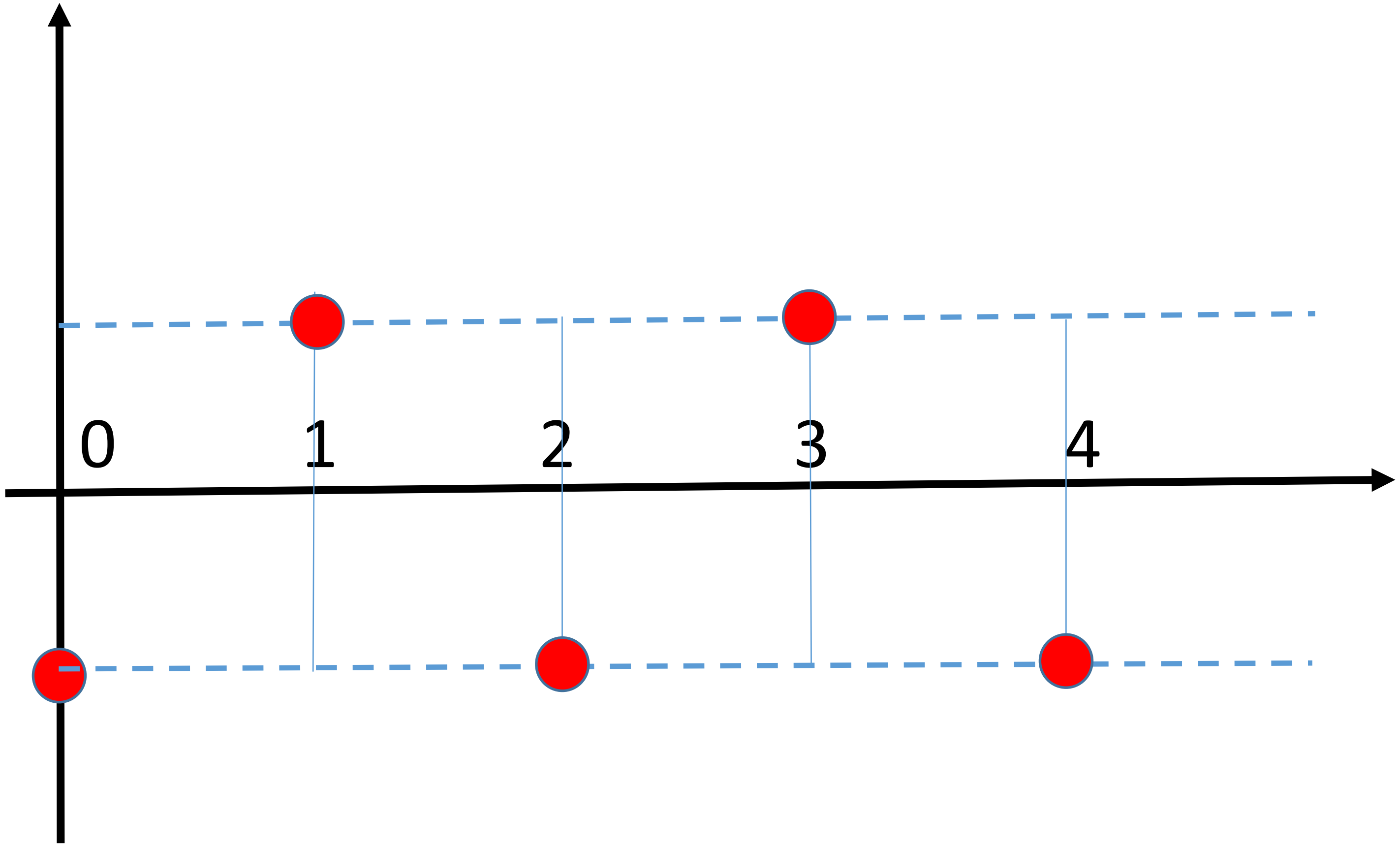
$u_3 = q \times u_2 = (-1) \times u_0 = -u_0$

$u_4 = q \times u_3 = (-1) \times (-u_0) = u_0$

Suite alternativement croissante et décroissante

Pas de limite ( les termes $u_n$ ne se stabilisent pas ).

$u_{n+1} = u_n \times q$ $u_0 < 0$ 2ème cas $q < 0$

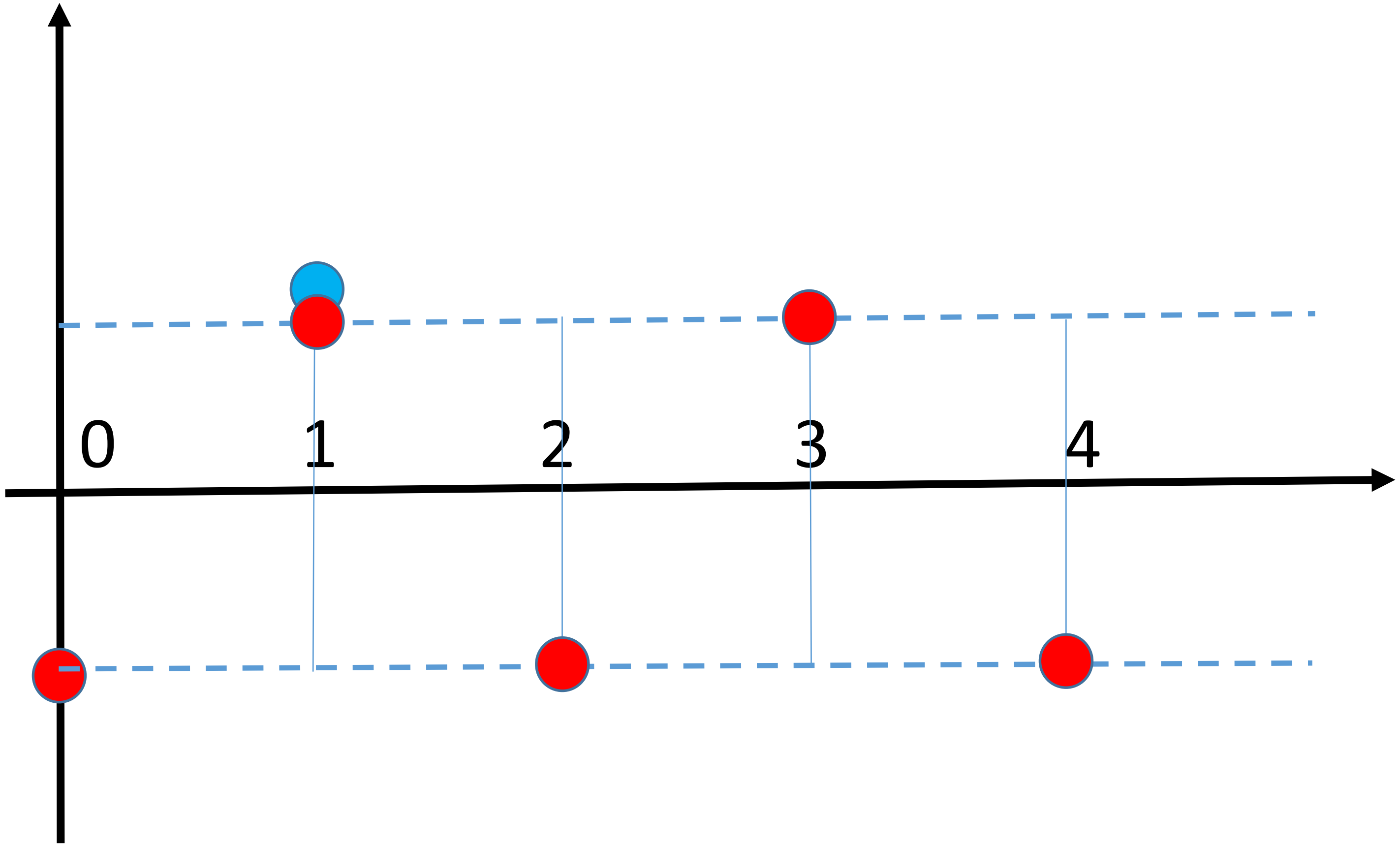



$q = -1$

6ème cas : $q < -1$

par ex. tracé pour $q = -1{,}1$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{ème}$ cas $q < 0$

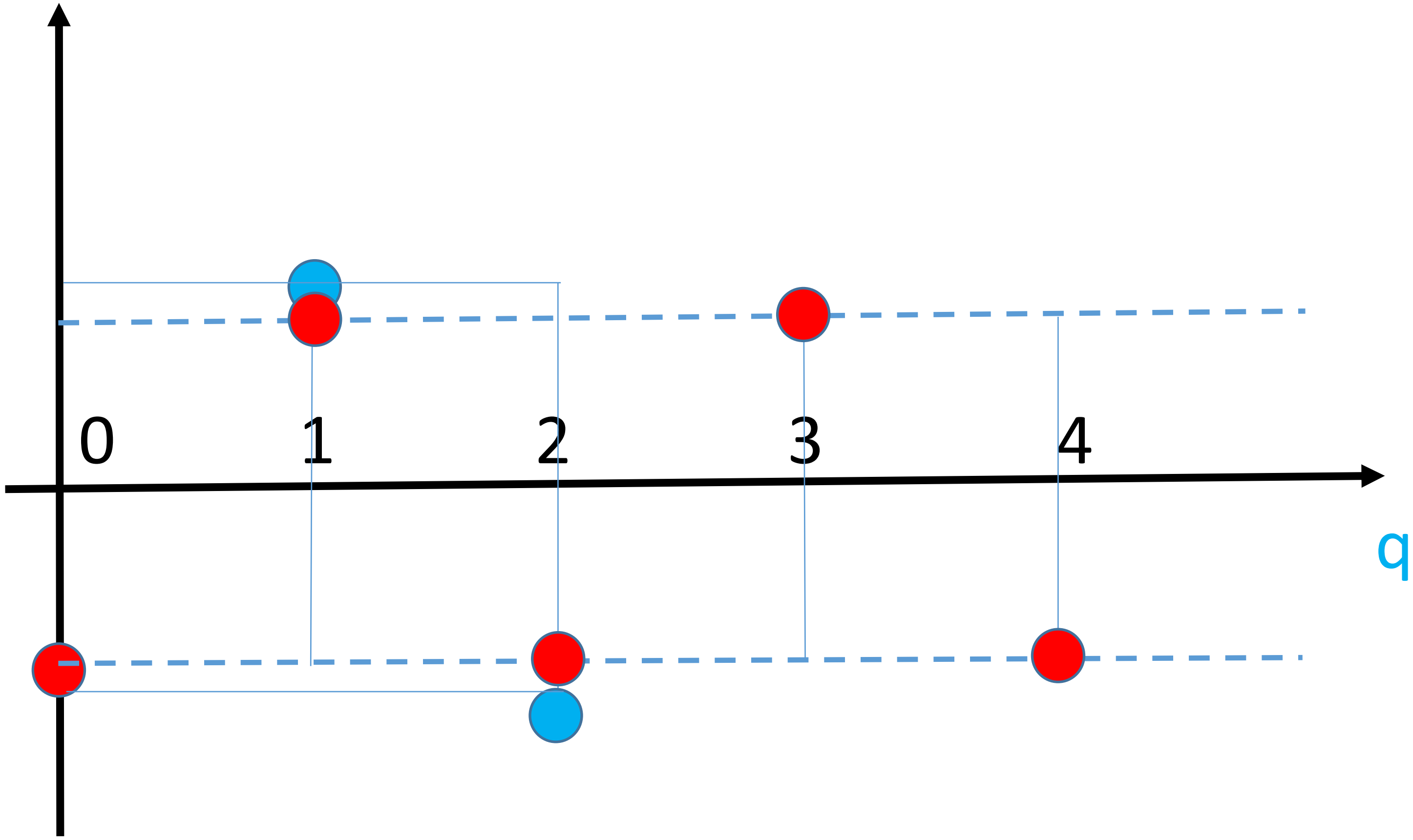



$q = -1$

$q < -1$

$q \times u_0 > -1 \times u_0$ $u_1 > -u_0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$



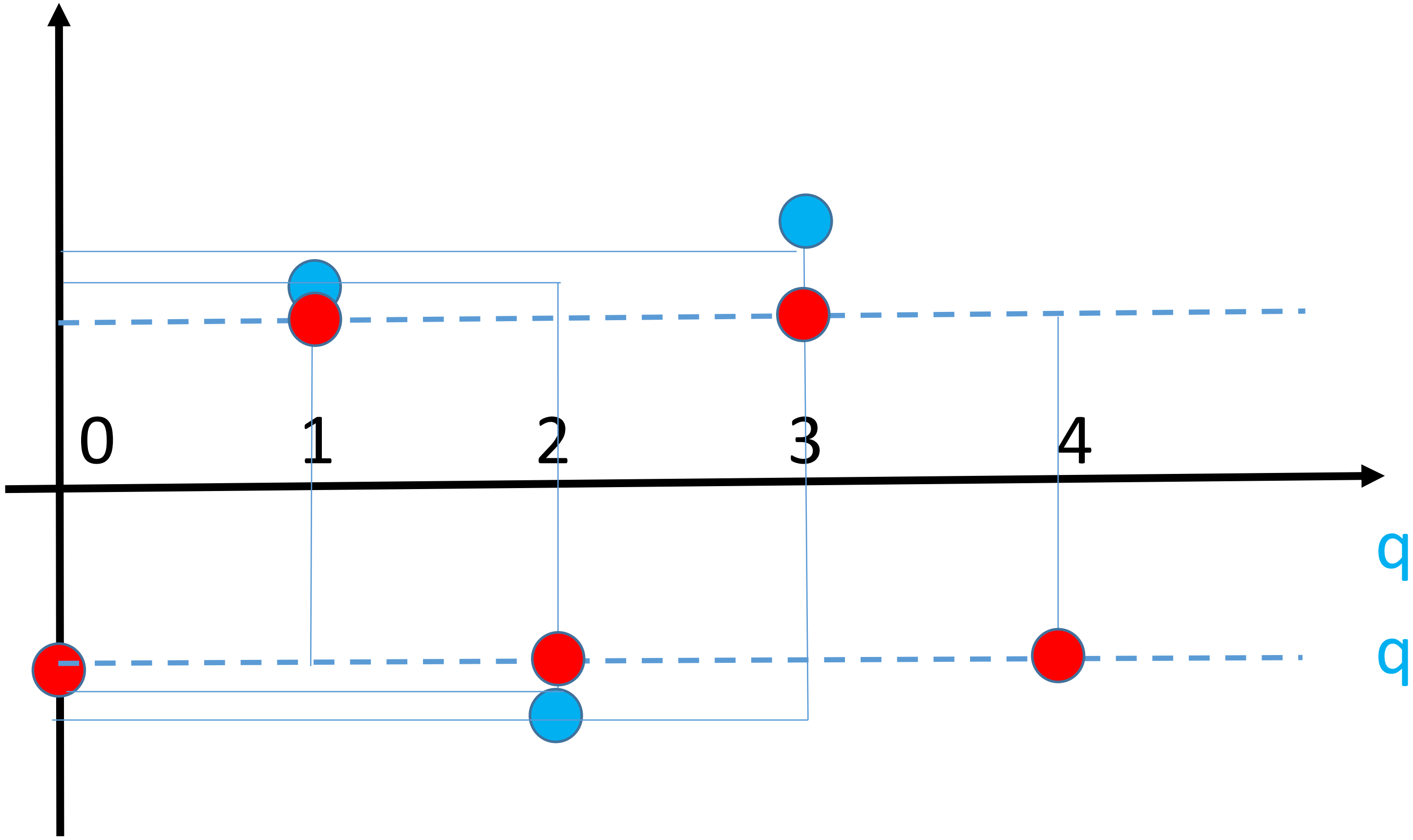

$q = -1$

$q < -1$

$q \times u_0 > -1 \times u_0$ $u_1 > -u_0$

$q \times u_1 < q \times (-u_0)$ $u_2 < -u_1$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad 2^{\text{ème}}$ cas $q < 0$




$q = -1$

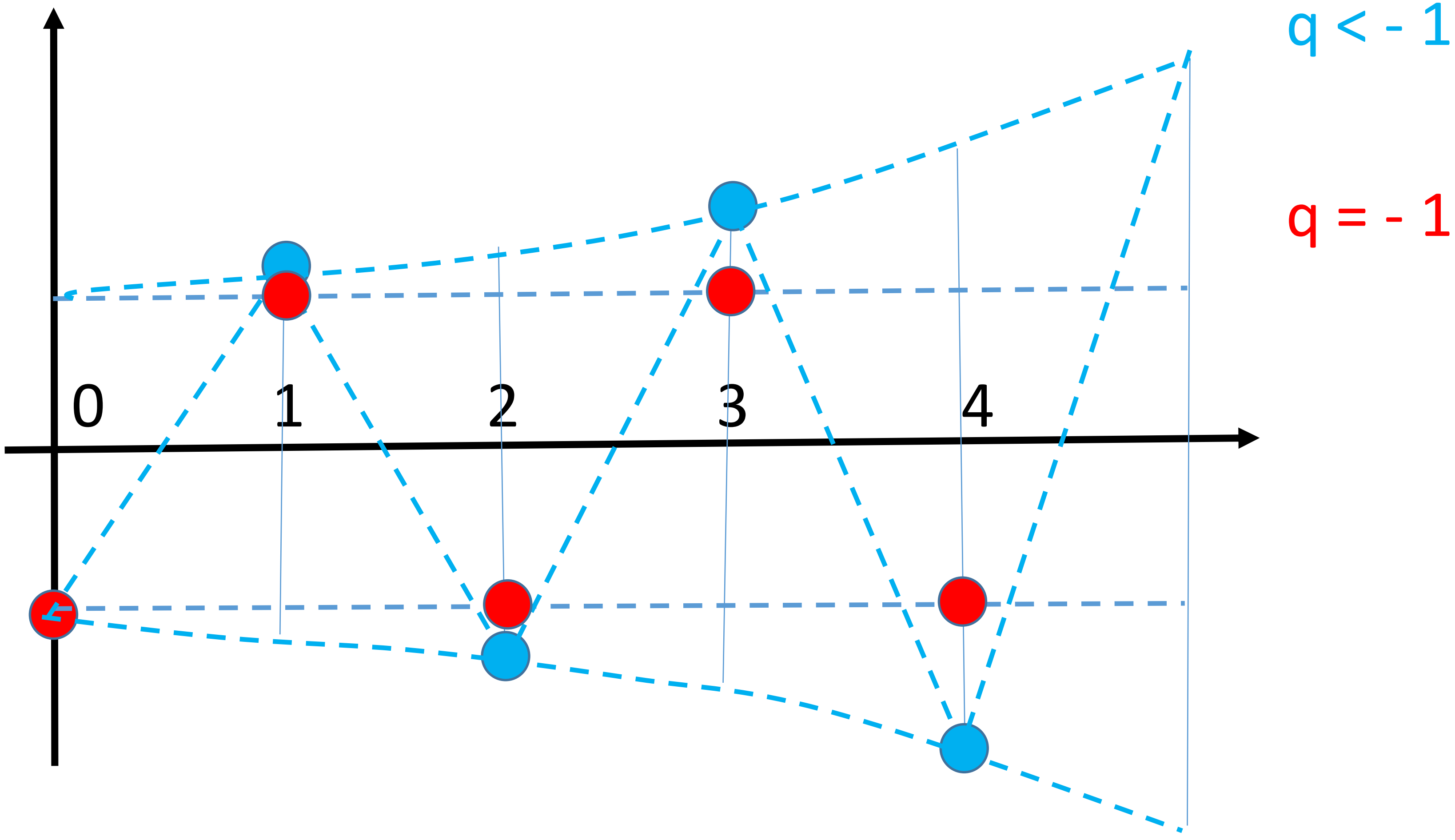
$q < -1$

$q \times u_0 > -1 \times u_0 \qquad u_1 > -u_0$

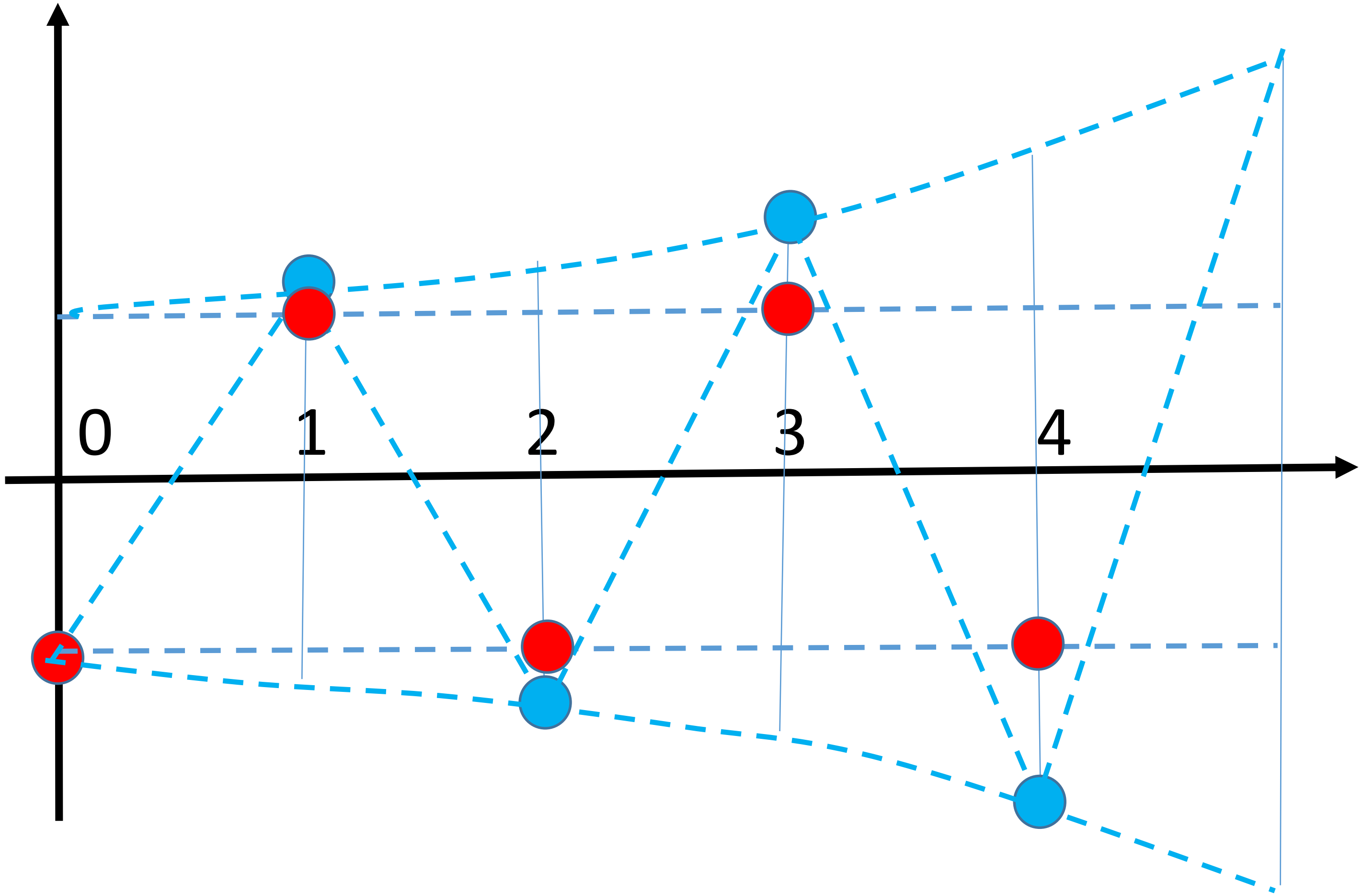
$q \times u_1 < q \times (-u_0) \qquad u_2 < -u_1$

$q \times u_2 > q \times (-u_1) \qquad u_3 > -u_2$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ $2^{\text{ème}}$ cas $q < 0$



$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{ème}$ cas $q < 0$



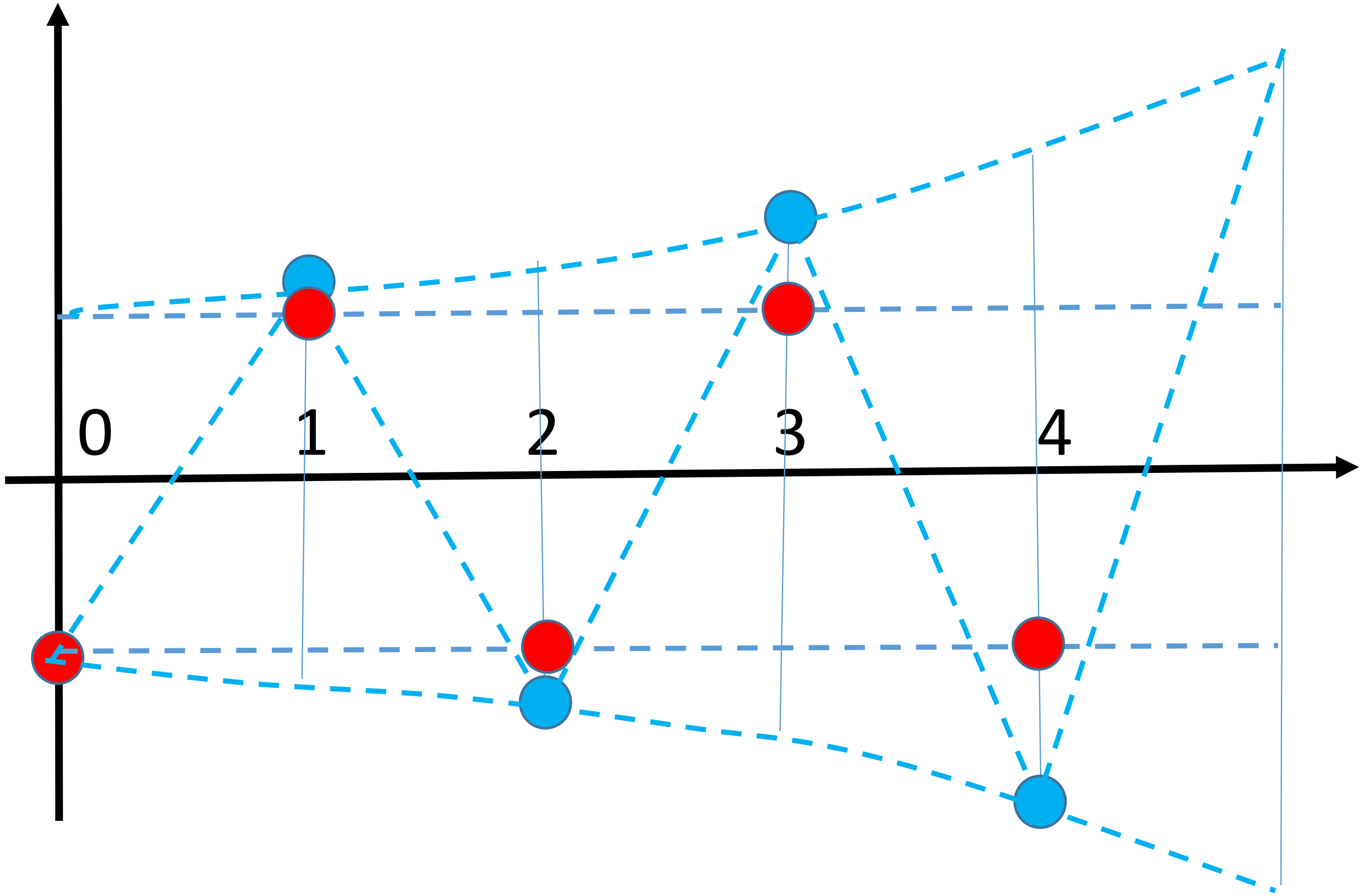

$q < -1$

$q = -1$

Suite alternativement croissante et décroissante

Pas de limite ( les termes $u_n$ ne se stabilisent pas )

$u_{n+1} = u_n \times q$ $\quad u_0 < 0 \quad$ $2^{\text{ème}}$ cas $q < 0$



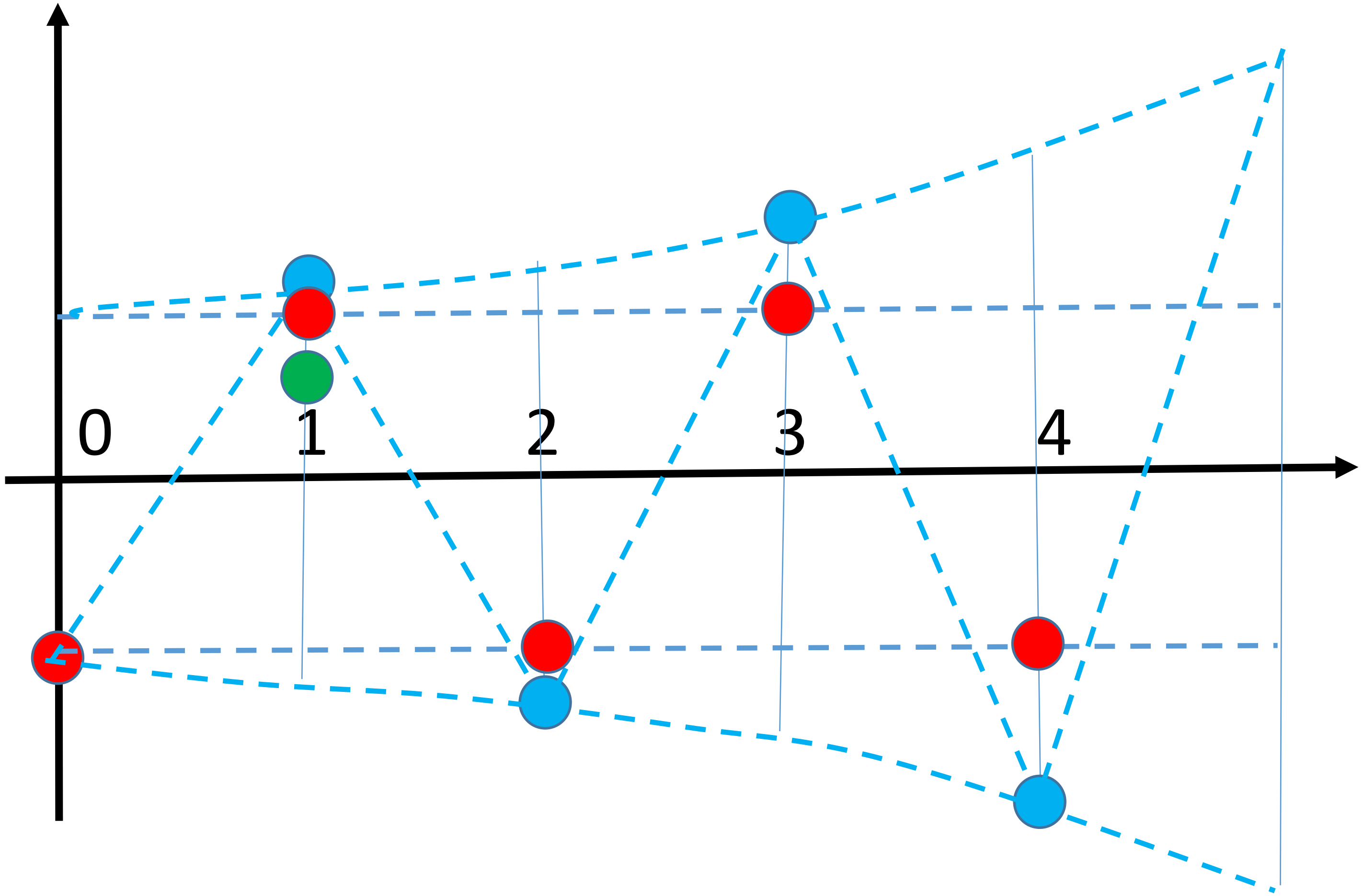

$q < -1$

$q = -1$

$7^{\text{ème}}$ cas : $-1 < q < 0$

Par ex. tracé pour $q = -0{,}7$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$




$q < -1$

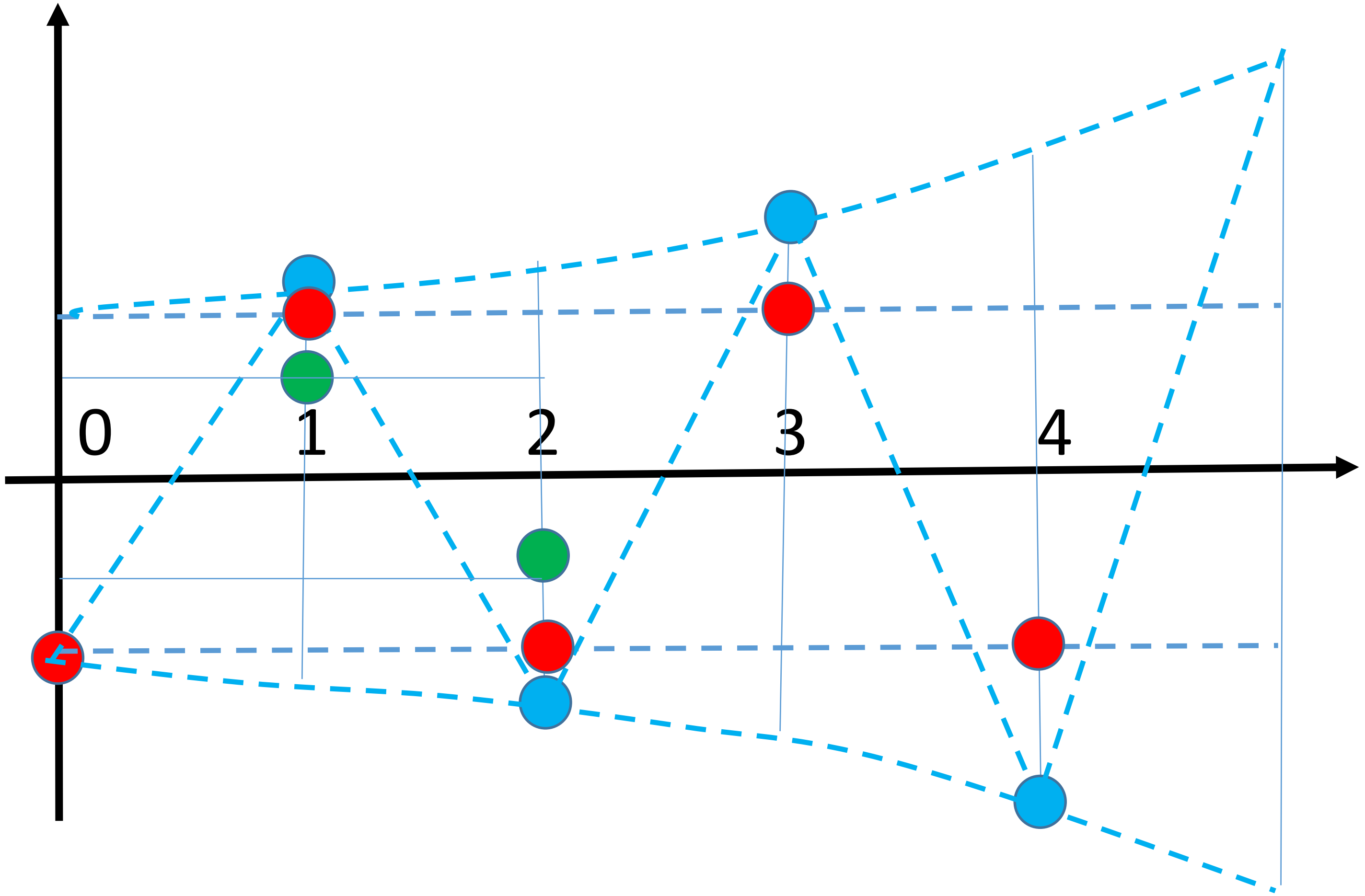
$q = -1$

$-1 < q < 0$

$-1 \times u_0 > q \times u_0 > 0 \times u_0$

$-u_0 > u_1 > 0$

$u_{n+1} = u_n \times q$ $\quad u_0 < 0$ $\quad$ 2ème cas $q < 0$




$q < -1$

$q = -1$

$-1 < q < 0$

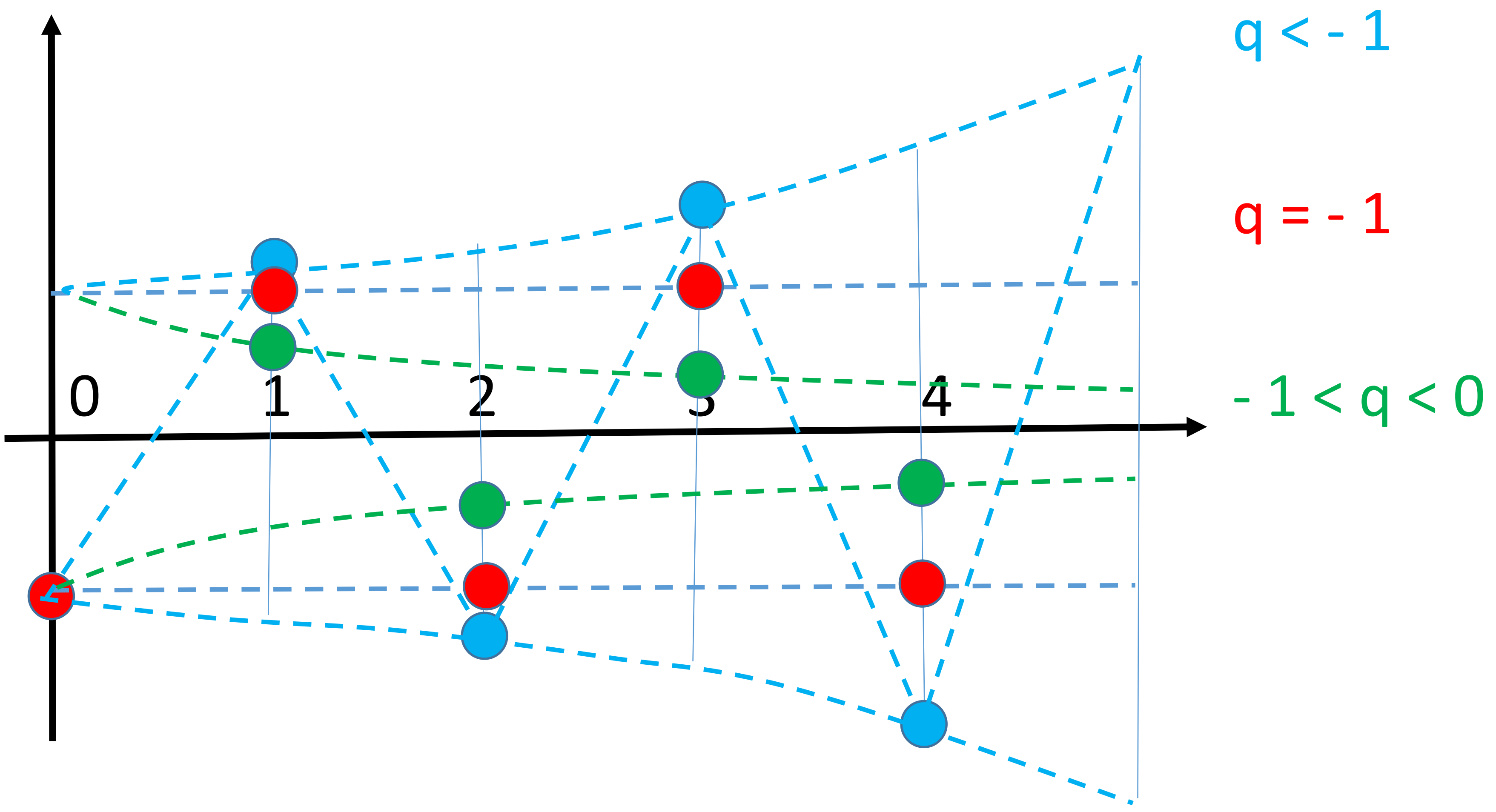
$-1 \times u_0 > q \times u_0 > 0 \times u_0$

$-u_0 > u_1 > 0$

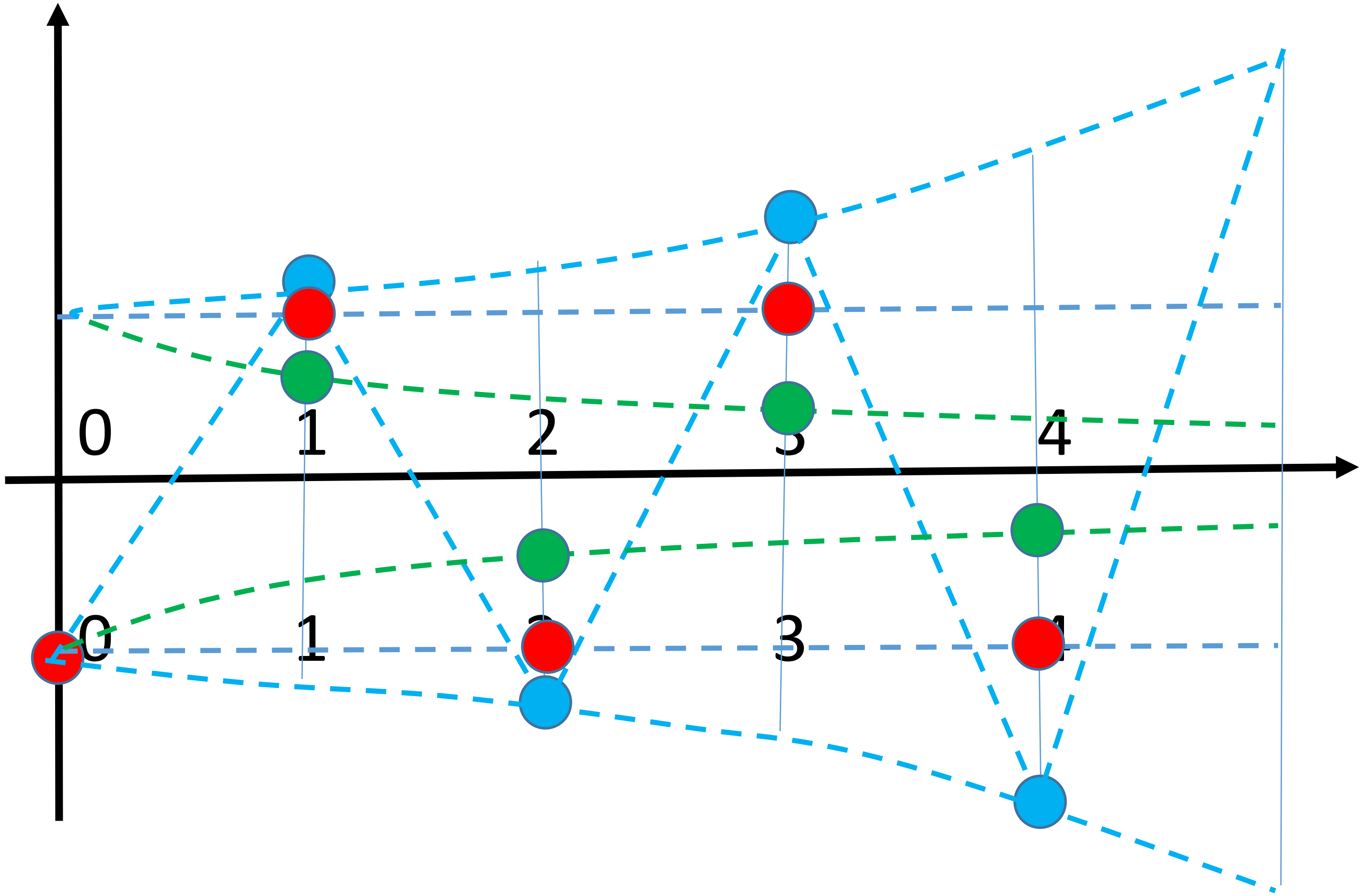
$-u_0 \times q < u_1 \times q < 0 \times q$

$-u_1 < u_2 < 0$

$u_{n+1} = u_n \times q$ $u_0 < 0$ $2^{\text{ème}}$ cas $q < 0$



$u_{n+1} = u_n \times q$ $u_0 < 0$ 2ème cas $q < 0$




$q < -1$

$q = -1$

$-1 < q < 0$

Suite alternativement croissante et décroissante

Limite : lorsque $n \to +\infty$

$u_n \to 0$