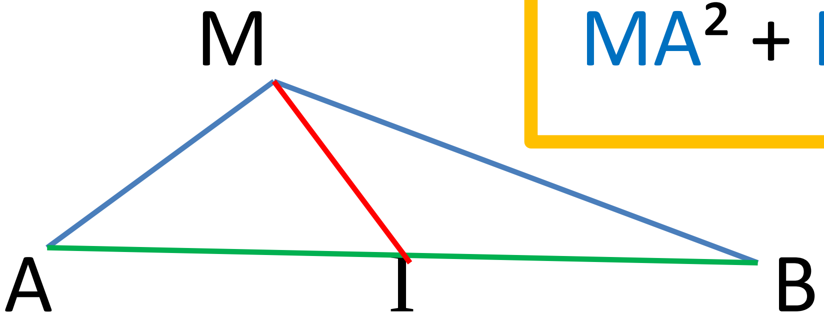
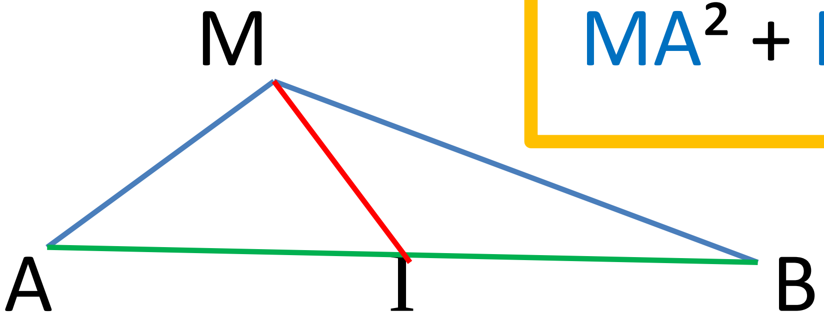
III Théorème de la médiane



$$MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2$$

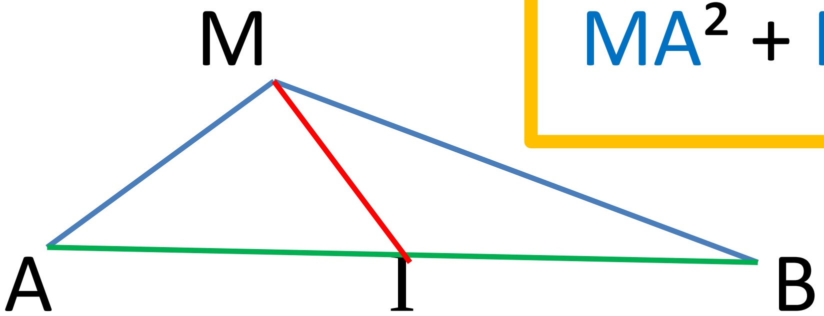
III Théorème de la médiane



$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \frac{1}{2}\,AB^2}$$

Démonstration : $MA^2 + MB^2 =$

III Théorème de la médiane



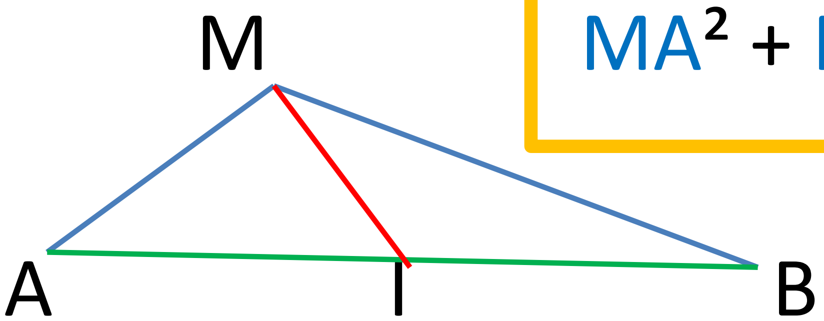
$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2}$$

Démonstration : $MA^2 + MB^2 = \overrightarrow{MA}^{\,2} + \overrightarrow{MB}^{\,2}$

$=$

III Théorème de la médiane

$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2}$$

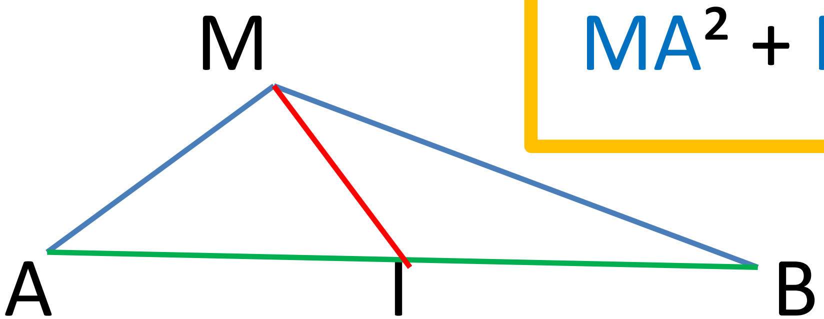


Démonstration : $MA^2 + MB^2 = \overrightarrow{MA}^{\,2} + \overrightarrow{MB}^{\,2}$

$= (\overrightarrow{MI} + \overrightarrow{IA})^2 + (\overrightarrow{MI} + \overrightarrow{IB})^2$

$=$

III Théorème de la médiane



$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2}$$

Démonstration : $MA^2 + MB^2 = \overrightarrow{MA}^{\,2} + \overrightarrow{MB}^{\,2}$

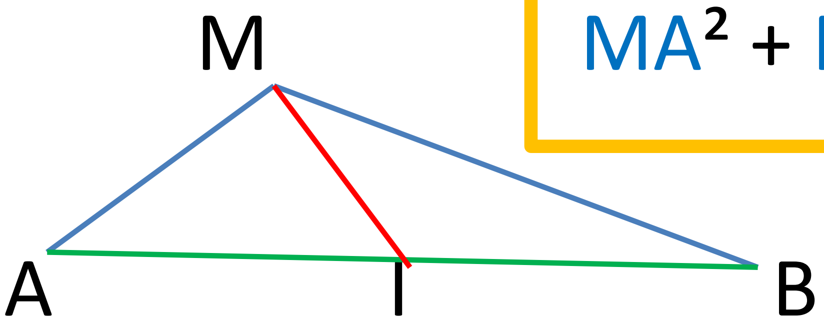
$= (\overrightarrow{MI} + \overrightarrow{IA})^2 + (\overrightarrow{MI} + \overrightarrow{IB})^2$

$= \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IA} + \overrightarrow{IA}^{\,2} + \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IB} + \overrightarrow{IB}^{\,2}$

$=$

III Théorème de la médiane

$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2}$$



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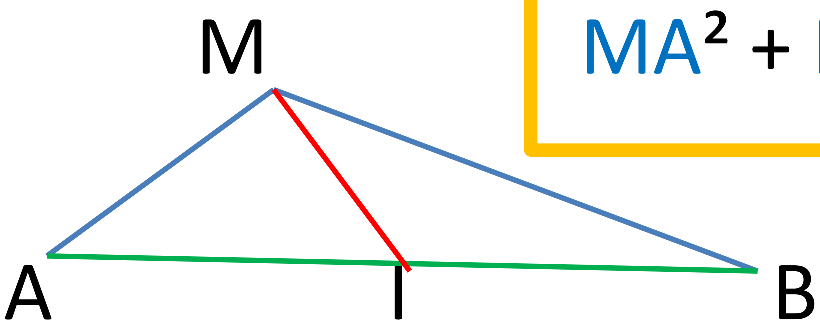
$=$

III Théorème de la médiane



$$MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2$$

Démonstration : $MA^2 + MB^2 = \overrightarrow{MA}^{\,2} + \overrightarrow{MB}^{\,2}$

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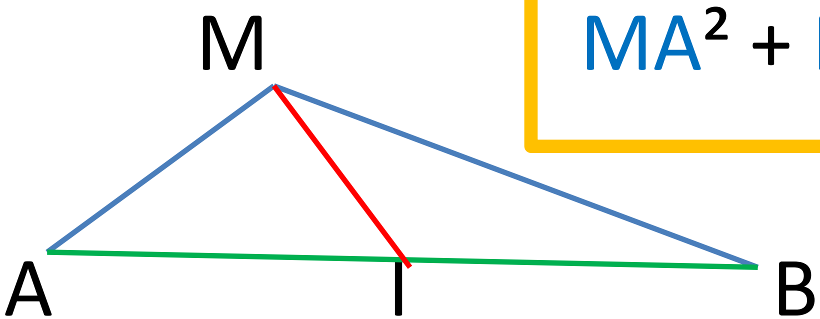
$= \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IA} + \overrightarrow{IA}^{\,2} + \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IB} + \overrightarrow{IB}^{\,2}$

$= 2\,\overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,(\overrightarrow{IA} + \overrightarrow{IB}) + (-\tfrac{1}{2}\,\overrightarrow{AB})^2 + (\tfrac{1}{2}\,\overrightarrow{AB})^2$

$=$

III Théorème de la médiane



$$\boxed{MA^2 + MB^2 = 2\,MI^2 + \tfrac{1}{2}\,AB^2}$$

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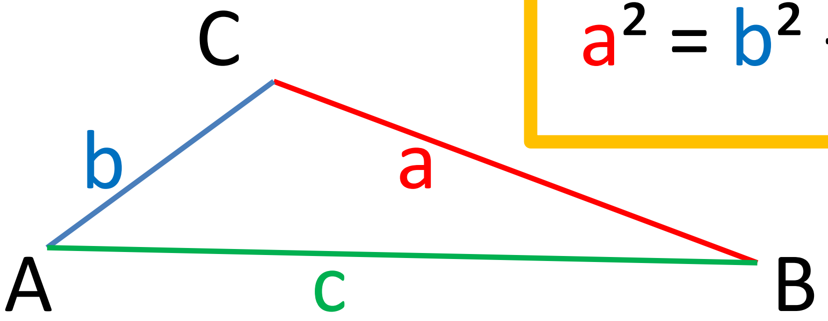
$= \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IA} + \overrightarrow{IA}^{\,2} + \overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,\overrightarrow{IB} + \overrightarrow{IB}^{\,2}$

$= 2\,\overrightarrow{MI}^{\,2} + 2\,\overrightarrow{MI}\,.\,(\overrightarrow{IA} + \overrightarrow{IB}) + (-\tfrac{1}{2}\,\overrightarrow{AB})^2 + (\tfrac{1}{2}\,\overrightarrow{AB})^2$

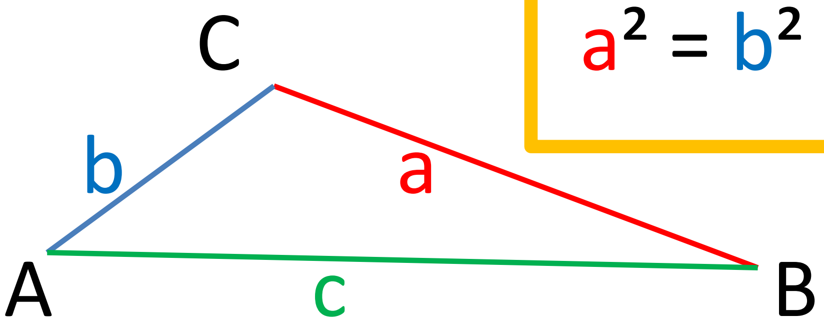
$= 2\,MI^2 + 2\,\overrightarrow{MI}\,.\,(\overrightarrow{0}) + \tfrac{1}{4}\,AB^2 + \tfrac{1}{4}\,AB^2$

IV Formule d'Al-Kashi

$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$



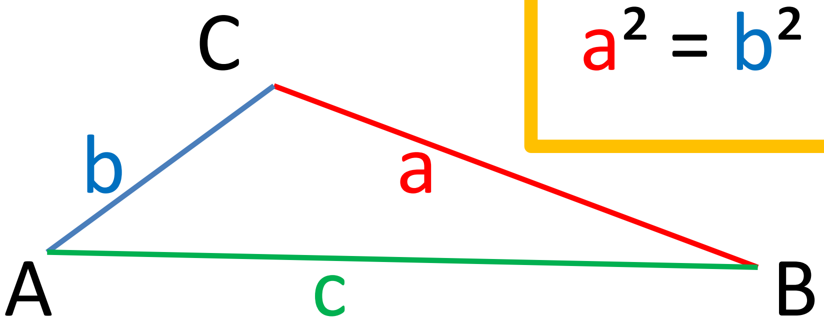
IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

Démonstration : $a^2 =$

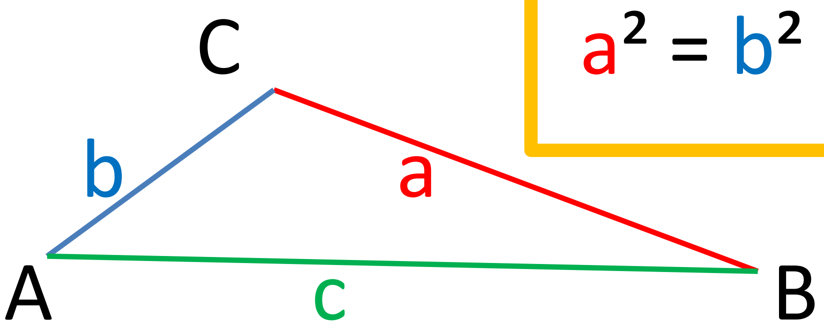
IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

Démonstration : $a^2 = BC^2 =$

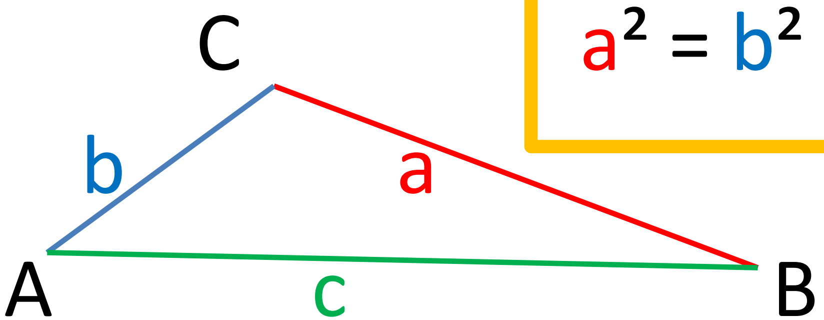
IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} =$

IV Formule d'Al-Kashi



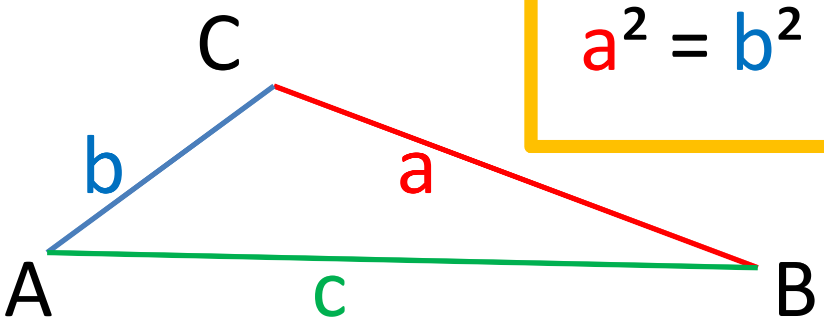
$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$

Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} = (\overrightarrow{BA} + \overrightarrow{AC})^2$

$= \overrightarrow{BA}^{\,2} + 2\,\overrightarrow{BA}\,.\,\overrightarrow{AC} + \overrightarrow{AC}^{\,2}$

$=$

IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} = (\,\overrightarrow{BA} + \overrightarrow{AC}\,)^2$

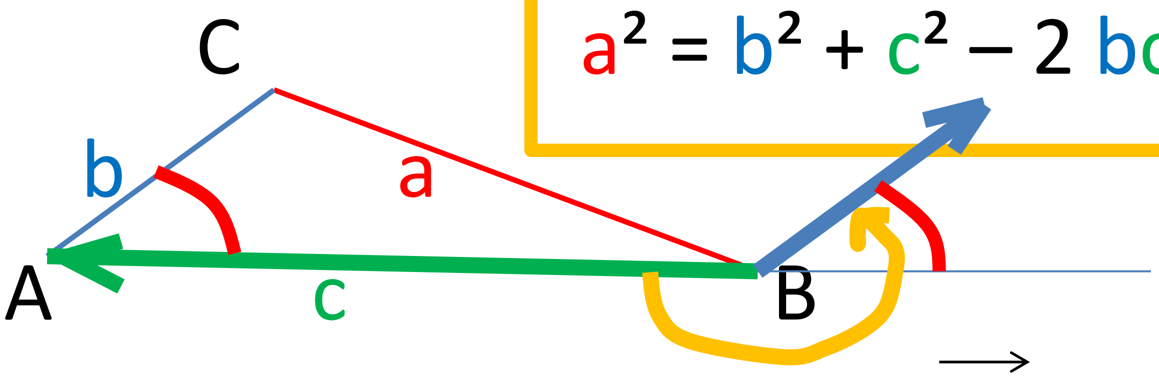
$= \overrightarrow{BA}^{\,2} + 2\,\overrightarrow{BA}\,.\,\overrightarrow{AC} + \overrightarrow{AC}^{\,2}$

$= BA^2 + 2\,BA \times AC \times \cos(\,\overrightarrow{BA}\,;\,\overrightarrow{AC}\,) + AC^2$

$=$

IV Formule d'Al-Kashi

$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$



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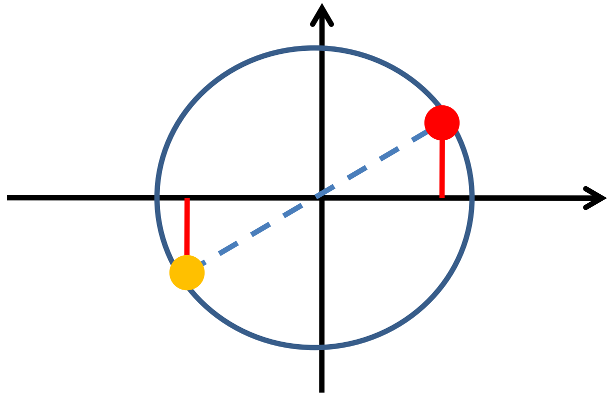
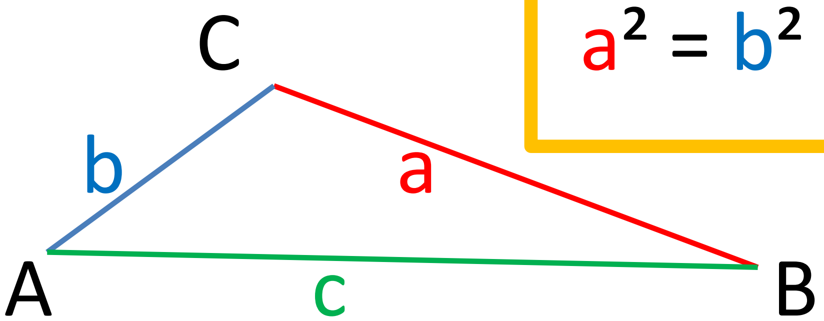
$= c^2 + 2\,c\,b\,(\,-\cos\widehat{A}\,) + b^2$



IV Formule d'Al-Kashi

$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$



Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} = (\,\overrightarrow{BA} + \overrightarrow{AC}\,)^2$

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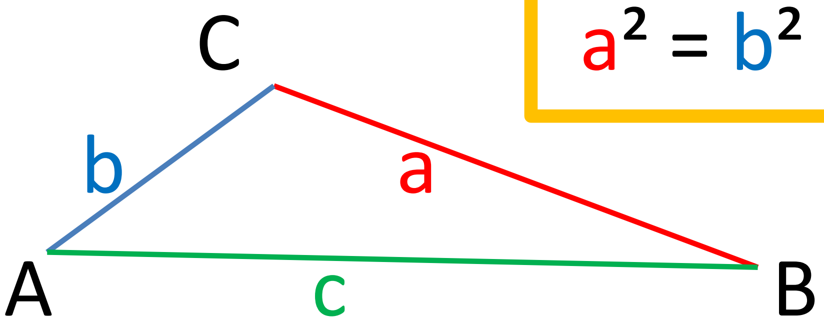
$= c^2 + 2\,c\,b\,(\,-\cos\widehat{A}\,) + b^2$

On a aussi $b^2 = a^2 + c^2 - 2\,ac\cos\widehat{B}$ et $c^2 =$ etc …

IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

appelée aussi …

Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} = (\,\overrightarrow{BA} + \overrightarrow{AC}\,)^2$

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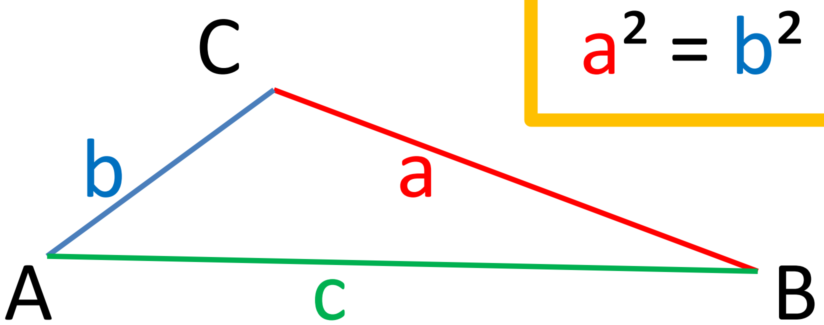
IV Formule d'Al-Kashi



$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$

appelée aussi

Pythagore généralisée.

Démonstration : $a^2 = BC^2 = \overrightarrow{BC}^{\,2} = (\,\overrightarrow{BA} + \overrightarrow{AC}\,)^2$

$= \overrightarrow{BA}^{\,2} + 2\,\overrightarrow{BA}\,.\,\overrightarrow{AC} + \overrightarrow{AC}^{\,2}$

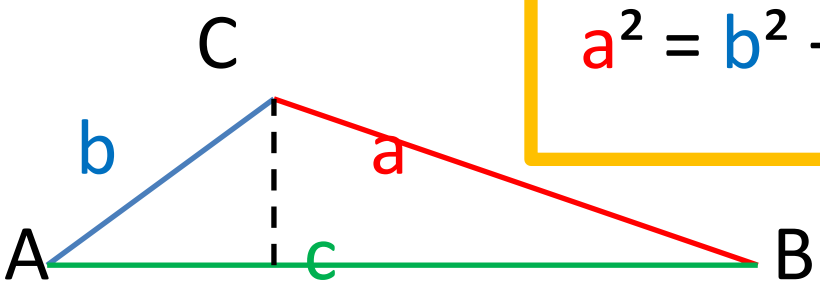
$= BA^2 + 2\,BA \times AC \times \cos(\,\overrightarrow{BA}\,;\,\overrightarrow{AC}\,) + AC^2$

$= c^2 + 2\,c\,b\,(\,-\cos\widehat{A}\,) + b^2$

On a aussi $b^2 = a^2 + c^2 - 2\,ac\cos\widehat{B}$

et $c^2 = a^2 + b^2 - 2\,ab\cos\widehat{C}$

IV Formule d'Al-Kashi



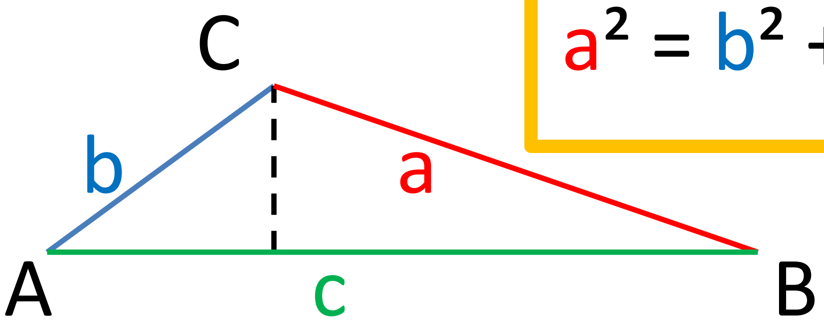
$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$

On obtient : $S = \frac{1}{2}\,c\,h = \frac{1}{2}\,c\,(\,b\sin\widehat{A}\,)$

Car $S = \frac{1}{2}$ base × hauteur

et $\sin\widehat{A} = \dfrac{\text{opposé}}{\text{hypoténuse}} = \dfrac{h}{b}$

IV Formule d'Al-Kashi

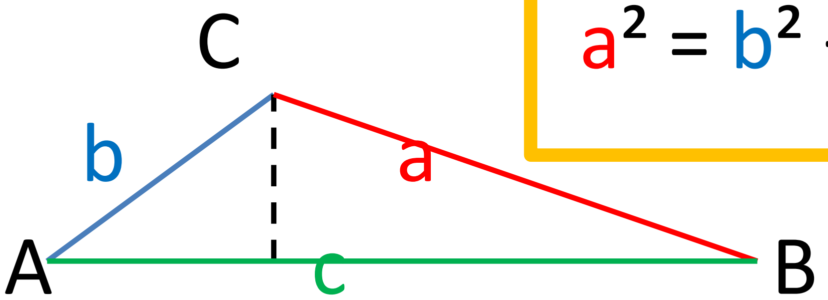


$$\boxed{a^2 = b^2 + c^2 - 2\,bc \cos \widehat{A}}$$

On obtient aussi : S = ½ c h = ½ c (b sin $\widehat{A}$)

et aussi S = ½ c (a sin $\widehat{B}$) et S = ½ a (b sin $\widehat{C}$)

IV Formule d'Al-Kashi



$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$

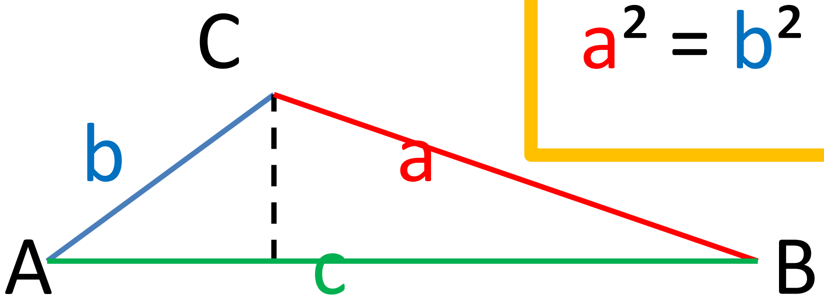
On obtient aussi : $S = \frac{1}{2}\,c\,h = \frac{1}{2}\,c\,(\,b\sin\widehat{A}\,)$

et aussi $S = \frac{1}{2}\,c\,(\,a\sin\widehat{B}\,)$ et $S = \frac{1}{2}\,a\,(\,b\sin\widehat{C}\,)$

qui donne $\dfrac{\dots}{2S} = \dots$

IV Formule d'Al-Kashi

$$a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}$$

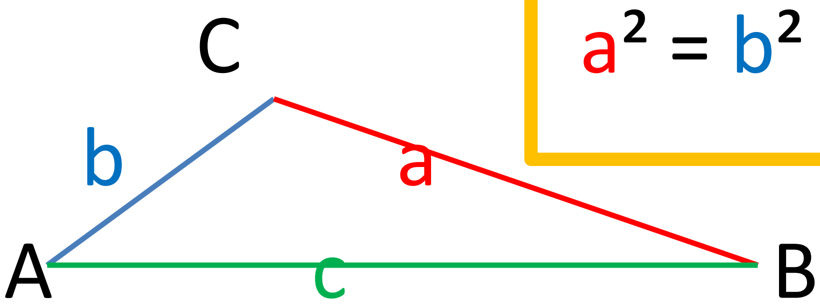


On obtient aussi : $S = \frac{1}{2}\, c\, h = \frac{1}{2}\, c\, (\, b \sin \widehat{A}\,)$

et aussi $S = \frac{1}{2}\, c\, (\, a \sin \widehat{B}\,)$ et $S = \frac{1}{2}\, a\, (\, b \sin \widehat{C}\,)$

qui donne $$\frac{abc}{2S} = \frac{a}{\sin \widehat{A}} = \frac{b}{\sin \widehat{B}} = \frac{c}{\sin \widehat{C}}$$

IV Formule d'Al-Kashi



$$\boxed{a^2 = b^2 + c^2 - 2\,bc\cos\widehat{A}}$$

On obtient aussi : $S = ½\, c\, h = ½\, c\,(\, b \sin\widehat{A}\,)$

et aussi $S = ½\, c\,(\, a \sin\widehat{B}\,)$ et $S = ½\, a\,(\, b \sin\widehat{C}\,)$

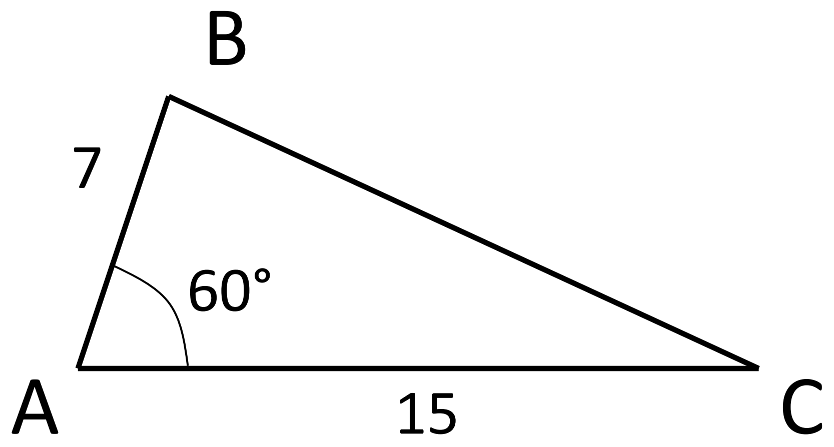
qui donne $$\frac{abc}{2S} = \boxed{\frac{a}{\sin\widehat{A}} = \frac{b}{\sin\widehat{B}} = \frac{c}{\sin\widehat{C}}}$$

appelée Formule des sinus

Application :

Déterminez la longueur

de la médiane issue de A du triangle suivant :

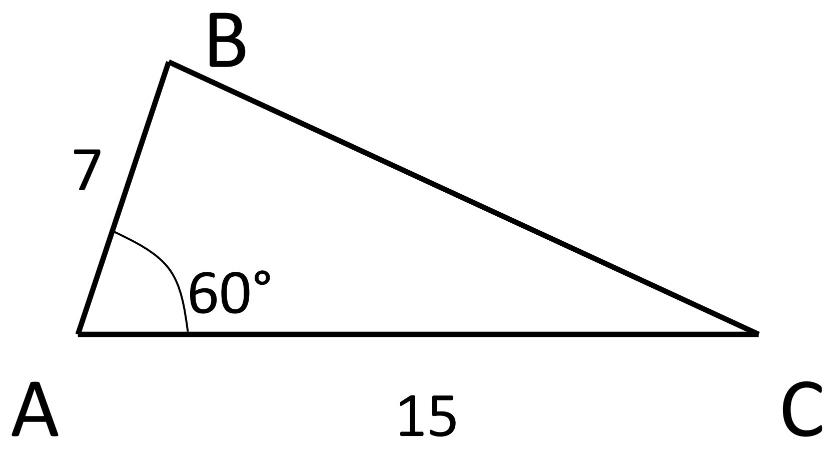


Déterminez la médiane issue de A.

Méthode :

On ne peut utiliser tout de suite le théorème de la médiane car il manque le 3ème côté.

Il sera déterminé avec le théorème d'Al-Kashi.



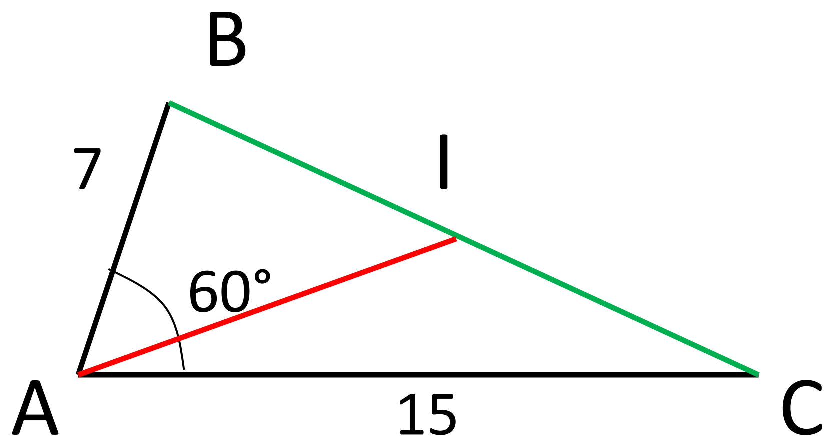
On cherche AI

Théorème de la médiane : $AB^2 + AC^2 = 2\ AI^2 + \frac{1}{2}\ BC^2$

Il nous faut connaître BC

Formule d'Al-Kashi : $BC^2 = AB^2 + AC^2 - 2\ AB\ AC \cos \widehat{A}$

$= 7^2 + 15^2 - 2\ (7)\ (15)\ \frac{1}{2} = 169 = 13^2$ donc BC $= 13$



On cherche AI

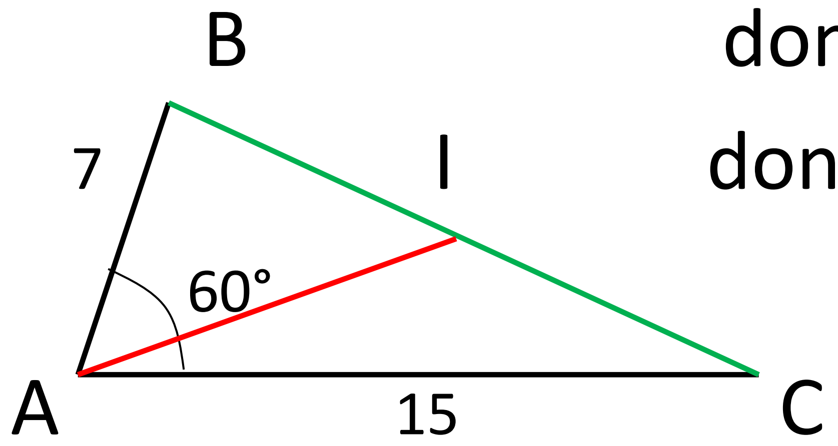
Théorème de la médiane : $AB^2 + AC^2 = 2\,AI^2 + \frac{1}{2}\,BC^2$

Il nous faut connaître BC

Formule d'Al-Kashi : $BC^2 = AB^2 + AC^2 - 2\,AB\,AC\cos\widehat{A}$

$= 7^2 + 15^2 - 2\,(7)\,(15)\,\frac{1}{2} = 169 = 13^2$ donc $BC = 13$

Th. de la médiane : $7^2 + 15^2 = 2\,AI^2 + \frac{1}{2}\,13^2$



donc $AI^2 = 94{,}75$

donc **AI =** $\sqrt{94{,}75}$