Exercice 9 :

1°) z_1 et z_2 ont comme formes **trigonométriques** respectives

$[\,2\,;\,11\pi/3\,]$ et $[\,\sqrt{2}\,;\,3\pi/4\,]$.

Déterminez leurs formes **algébriques**.

2°) z_3 et z_4 ont comme formes **algébriques** respectives $-\sqrt{2} - i\sqrt{2}$ et $\sqrt{6} + i\sqrt{2}$

Déterminez leurs formes **trigonométriques**.

3°) $z_5 = 4 + 4\sqrt{3}i$ et $z_6 = z_4 \times z_5$

Déterminez leurs formes **trigonométriques**.

Quelle conjecture peut-on faire ?

Exercice 9 : 1°)

z_1 de **forme** trigonométrique $z_1 = [\ 2\ ;\ 11\pi/3\ ]$

$a = r \cos \beta = 2\ (\cos 11\pi/3\ )$

$b = r \sin \beta = 2\ (\sin 11\pi/3\ )$

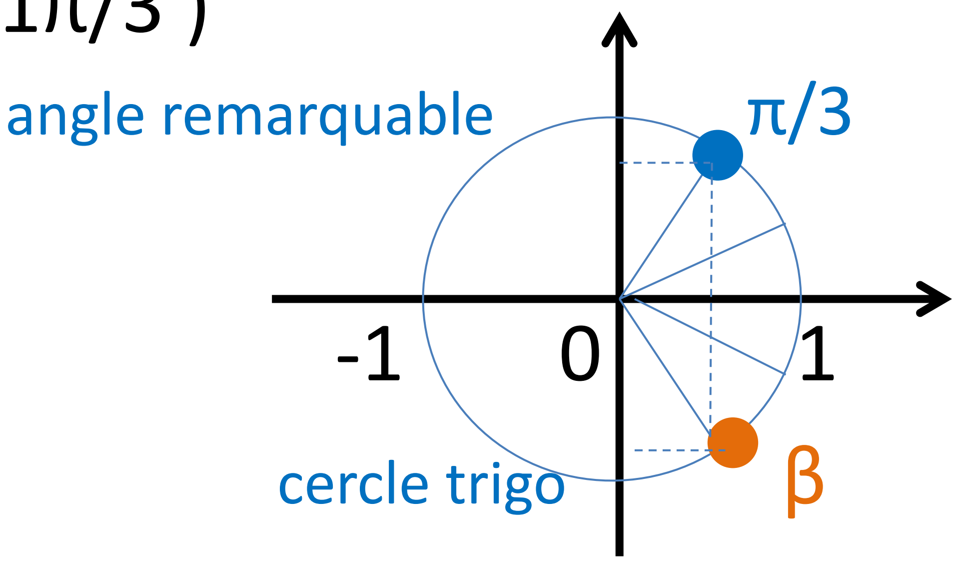
Exercice 9 : 1°)

z_1 de **forme** trigonométrique $z_1 = [\ 2\ ;\ 11\pi/3\ ]$

$a = r \cos \beta = 2\ (\cos 11\pi/3\ )$

$b = r \sin \beta = 2\ (\sin 11\pi/3\ )$

$11\pi/3 = 0 + 4\pi - \pi/3$

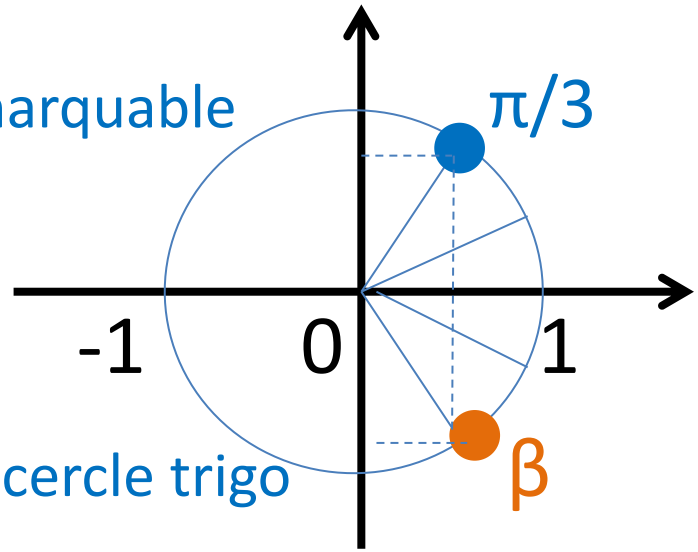


Exercice 9 : 1°)

z_1 de **forme** trigonométrique z_1 = [**2** ; **11π/3**]

a = r cos β = 2 (cos 11π/3)

b = r sin β = 2 (sin 11π/3)



11π/3 = 0 + 4π – π/3

a = 2 (+ 1 / 2) = 1

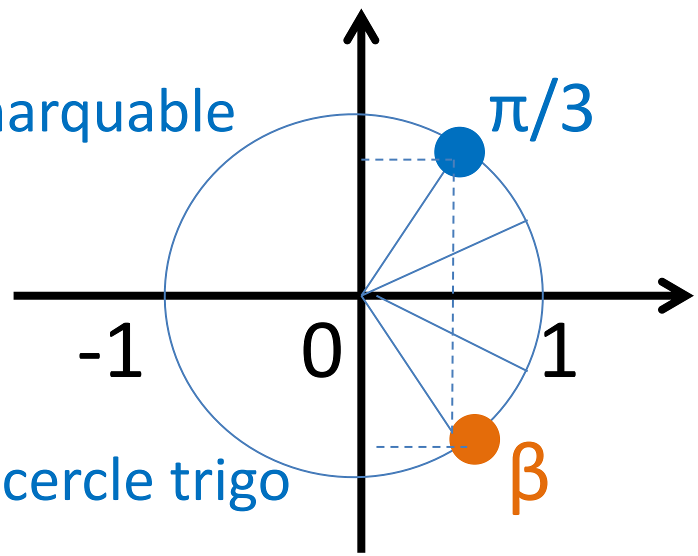
b = 2 (- (√3) / 2) = - √3

Exercice 9 : 1°)

z_1 de **forme** trigonométrique z_1 = [**2** ; **11π/3**]

a = r cos β = 2 (cos 11π/3)

b = r sin β = 2 (sin 11π/3)



11π/3 = 0 + 4π – π/3

a = 2 (+ 1 / 2) = 1

b = 2 (- (√3) / 2) = - √3

Z_1 = [**2** ; **11π/3**]

forme trigonométrique

Z_1 = 1 – i √3

forme algébrique

Exercice 9 : **1°)**

z_2 de **forme** trigonométrique z_2 = [√2 ; 3π/4]

a = r cos β = √2 (cos 3π/4)

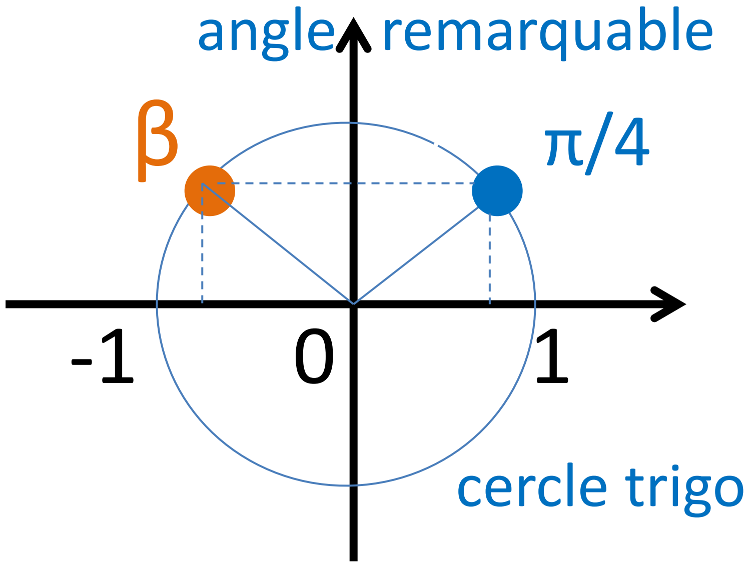
b = r sin β = √2 (cos 3π/4)

Exercice 9 : 1°)

z_2 de **forme** trigonométrique z_2 = [**√2** ; **3π/4**]

a = r cos β = √2 (cos 3π/4)

b = r sin β = √2 (cos 3π/4)

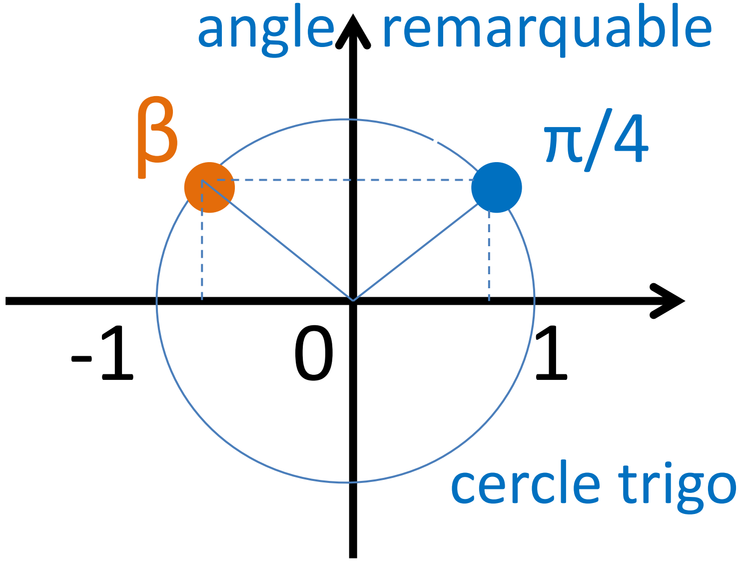


Exercice 9 : 1°)

z_2 de **forme** trigonométrique $z_2 = [\ \sqrt{2}\ ;\ 3\pi/4\ ]$

$a = r \cos \beta = \sqrt{2}\ (\cos 3\pi/4)$

$b = r \sin \beta = \sqrt{2}\ (\cos 3\pi/4)$



$a = \sqrt{2}\ (\ -\ (\sqrt{2})\ /\ 2\ ) = -\ 1$

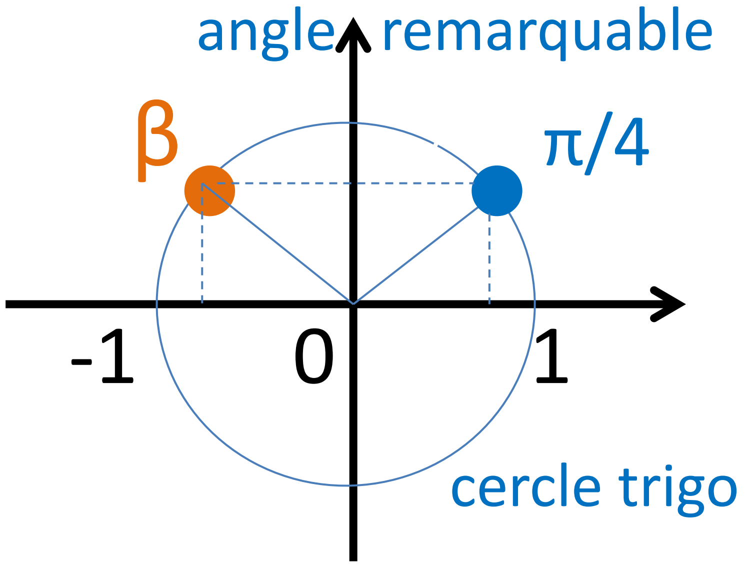
$b = \sqrt{2}\ (\ +\ (\sqrt{2})\ /\ 2\ ) = 1$

Exercice 9 : 1°)

z_2 de **forme** trigonométrique $z_2 = [\ \sqrt{2}\ ;\ 3\pi/4\ ]$

$a = r \cos \beta = \sqrt{2}\ (\cos 3\pi/4)$

$b = r \sin \beta = \sqrt{2}\ (\cos 3\pi/4)$



$a = \sqrt{2}\ (-\ (\sqrt{2})\ /\ 2\ ) = -\ 1$

$b = \sqrt{2}\ (+\ (\sqrt{2})\ /\ 2\ ) = 1$

$Z_2 = [\ \sqrt{2}\ ;\ 3\pi/4\ ]$ — forme trigonométrique

$Z_2 = -\ 1 + i$ — forme algébrique

Exercice 9 : 2°)

z_3 de **forme algébrique** $z_3 = -\sqrt{2} - i\sqrt{2}$

$r = |z|$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$$

Exercice 9 : 2°)

z_3 de **forme algébrique** $z_3 = -\sqrt{2} - i\sqrt{2}$

$r = |z|$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$$

$\beta = \arg(z)$

$$\begin{cases} \cos\beta = a/r = -(\sqrt{2})/2 \\ \sin\beta = b/r = -(\sqrt{2})/2 \end{cases}$$

Exercice 9 : 2°)

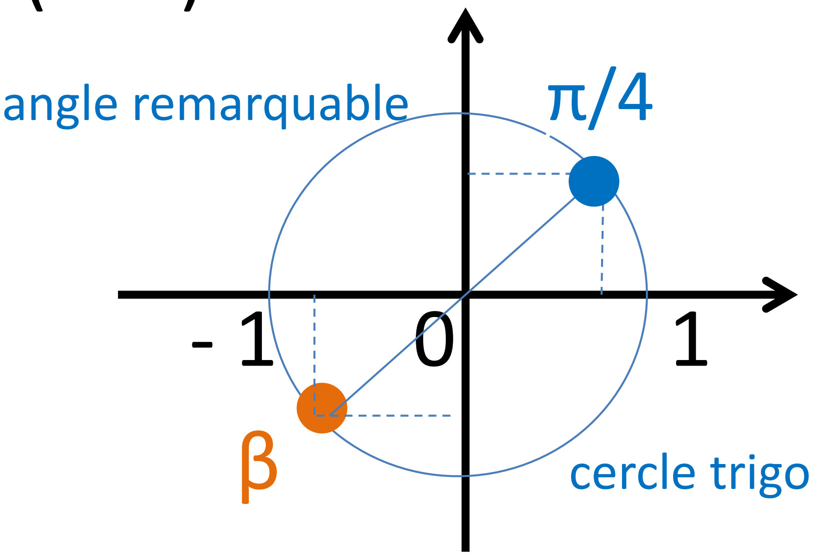
z_3 de **forme algébrique** $z_3 = -\sqrt{2} - i\sqrt{2}$

r = | z |

$$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$$

$\beta = \arg(z)$

$$\begin{cases} \cos \beta = a/r = -(\sqrt{2})/2 \\ \sin \beta = b/r = -(\sqrt{2})/2 \end{cases}$$



Exercice 9 : 2°)

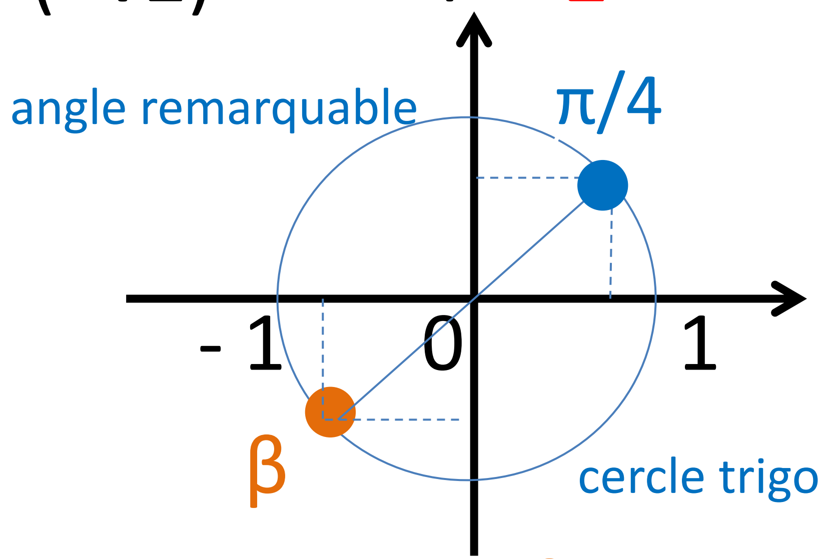
z_3 de **forme algébrique** $z_3 = -\sqrt{2} - i\sqrt{2}$

$r = |z|$

$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$

$\beta = \arg(z)$

$$\begin{cases} \cos\beta = a/r = -(\sqrt{2})/2 \\ \sin\beta = b/r = -(\sqrt{2})/2 \end{cases}$$



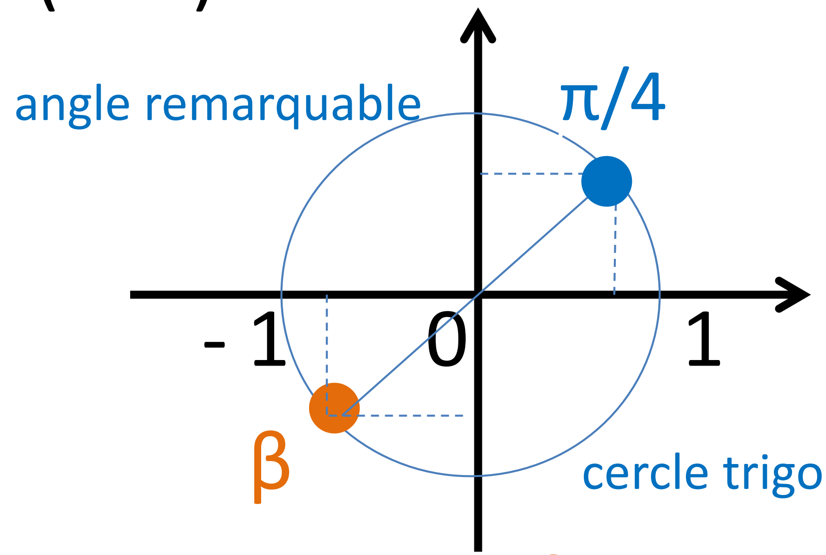
donc $\beta = 5\pi/4 + k2\pi$

Exercice 9 : 2°)

z_3 de **forme algébrique** $z_3 = -\sqrt{2} - i\sqrt{2}$

$r = | z |$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$$



$\beta = \arg(z)$

$$\begin{cases} \cos \beta = a/r = -(\sqrt{2}) / 2 \\ \sin \beta = b/r = -(\sqrt{2}) / 2 \end{cases}$$

donc $\beta = 5\pi/4 + k2\pi$

$Z_3 = -\sqrt{2} - i\sqrt{2}$

forme algébrique

$Z_3 = [\ 2\ ;\ 5\pi/4\ ]$

forme trigonométrique

Exercice 9 : 2°)

z_4 de **forme algébrique** $z_4 = \sqrt{6} + i\sqrt{2}$

$r = |z|$

$$r = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{8} = 2\sqrt{2}$$

Exercice 9 : 2°)

z_4 de **forme algébrique** $z_4 = \sqrt{6} + i\sqrt{2}$

$r = | z |$

$$r = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{8} = 2\sqrt{2}$$

$\beta = \arg(z)$

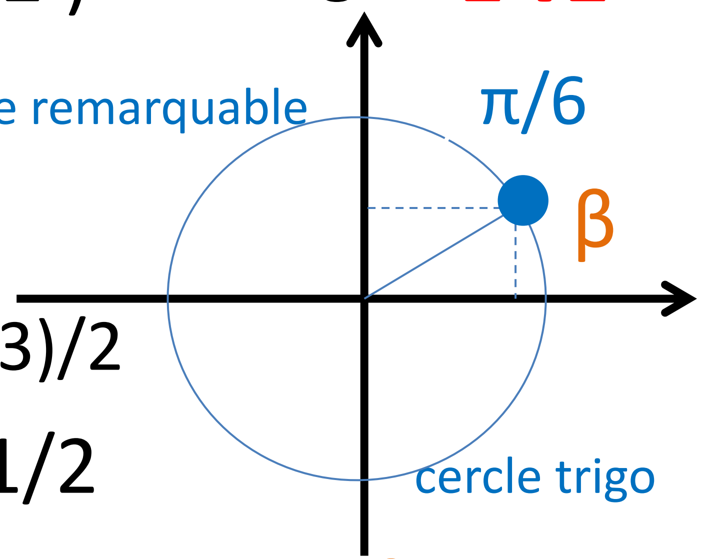
$$\begin{cases} \cos\beta = a/r = (\sqrt{6})/(2\sqrt{2}) = (\sqrt{3})/2 \\ \sin\beta = b/r = (\sqrt{2})/(2\sqrt{2}) = 1/2 \end{cases}$$

Exercice 9 : 2°)

z_4 de **forme algébrique** $z_4 = \sqrt{6} + i\sqrt{2}$

$r = | z |$

$$r = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{8} = 2\sqrt{2}$$



$\beta = \arg(z)$

$$\begin{cases} \cos\beta = a/r = (\sqrt{6})/(2\sqrt{2}) = (\sqrt{3})/2 \\ \sin\beta = b/r = (\sqrt{2})/(2\sqrt{2}) = 1/2 \end{cases}$$

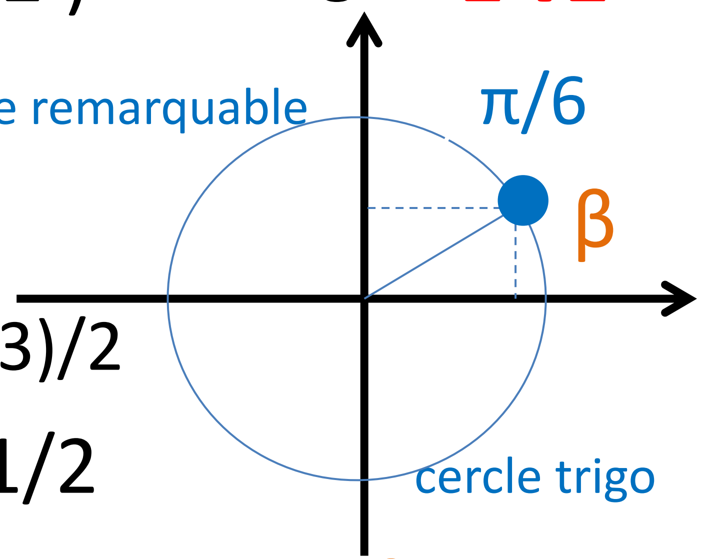
donc $\beta = \pi/6 + k2\pi$

Exercice 9 : 2°)

z_4 de **forme algébrique** $z_4 = \sqrt{6} + i\sqrt{2}$

$r = | z |$

$$r = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{8} = 2\sqrt{2}$$



$\beta = \arg(z)$

$$\begin{cases} \cos\beta = a/r = (\sqrt{6})/(2\sqrt{2}) = (\sqrt{3})/2 \\ \sin\beta = b/r = (\sqrt{2})/(2\sqrt{2}) = 1/2 \end{cases}$$

donc $\beta = \pi/6 + k2\pi$

$Z_4 = \sqrt{6} + i\sqrt{2}$

forme algébrique

$Z_4 = [\, 2\sqrt{2} \,;\, \pi/6 \,]$

forme trigonométrique

Exercice 9 :

3°) $z_5 = 4 + i\,4\sqrt{3}$ et $z_6 = z_4 \times z_5$

Déterminez leurs formes **trigonométriques**.

Quelle conjecture peut-on faire ?

Rappel question 2°) :

$Z_4 = \sqrt{6} + i\sqrt{2}$ $Z_4 = [\,2\sqrt{2}\,;\,\pi/6\,]$

Exercice 9 : 3°)

z_5 de **forme algébrique** $z_5 = 4 + i\,4\sqrt{3}$

$r = |\,z_5\,|$

$$r = \sqrt{a^2 + b^2} = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{64} = 8$$

Exercice 9 : 3°)

z_5 de **forme algébrique** $z_5 = 4 + i\,4\sqrt{3}$

$r = |z_5|$

$$r = \sqrt{a^2 + b^2} = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{64} = \mathbf{8}$$

$\beta = \arg(z_5)$

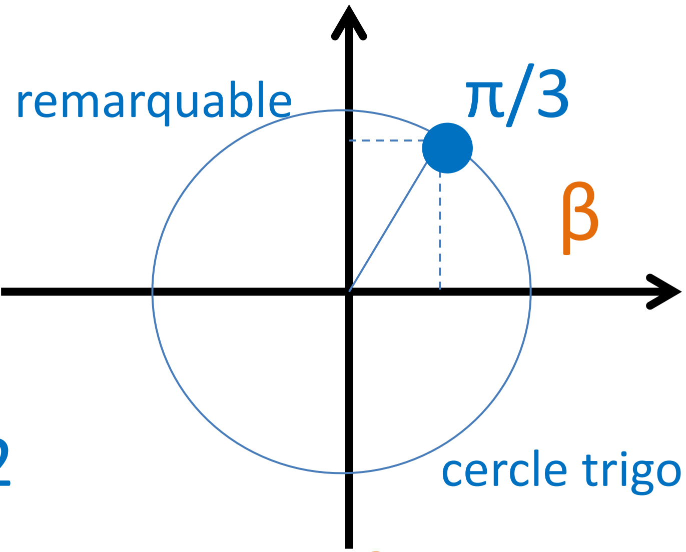
$$\begin{cases} \cos\beta = a/r = 4/8 = 1/2 \\ \sin\beta = b/r = (4\sqrt{3})/8 = (\sqrt{3})/2 \end{cases}$$

Exercice 9 : 3°)

z_5 de **forme algébrique** $z_5 = 4 + i\,4\sqrt{3}$

$r = |z_5|$

$$r = \sqrt{a^2 + b^2} = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{64} = 8$$



$\beta = \arg(z_5)$

$$\begin{cases} \cos \beta = a/r = 4/8 = 1/2 \\ \sin \beta = b/r = (4\sqrt{3})/8 = (\sqrt{3})/2 \end{cases}$$

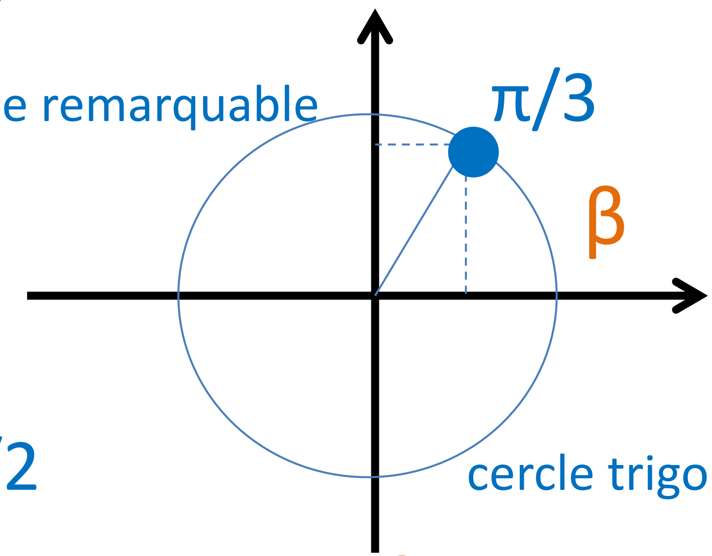
donc $\beta = \pi/3 + k2\pi$

Exercice 9 : 3°)

z_5 de **forme algébrique** $z_5 = 4 + i\,4\sqrt{3}$

$r = | z_5 |$

$$r = \sqrt{a^2 + b^2} = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{64} = 8$$



$\beta = \arg(z_5)$

$$\begin{cases} \cos\beta = a/r = 4/8 = 1/2 \\ \sin\beta = b/r = (4\sqrt{3})/8 = (\sqrt{3})/2 \end{cases}$$

donc $\beta = \pi/3 + k2\pi$

$Z_5 = 4 + i\,4\sqrt{3}$ forme algébrique

$Z_5 = [\,8\,;\,\pi/3\,]$ forme trigonométrique

3°)

$z_6 = z_4\ z_5 = (\ 4 + 4\ \sqrt{3}i\ ) \times (\ \sqrt{6} + i\ \sqrt{2}\ )$

$= 4\sqrt{6} + 4i\ \sqrt{18} + 4i\ \sqrt{2} + 4i^2\ \sqrt{6}$

$= 4\sqrt{6} + 4i\ 3\sqrt{2} + 4i\ \sqrt{2} + 4(-\ 1)\ \sqrt{6}$

$= 0 + i\ 16\sqrt{2}$

Exercice 9 : 3°)

z_6 de **forme algébrique** $z_6 = 16 i \sqrt{2}$

$r = | z_6 |$

$$r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (16\sqrt{2})^2} = \mathbf{16 \sqrt{2}}$$

Exercice 9 : 3°)

z_6 de **forme algébrique** $z_6 = 16\,i\,\sqrt{2}$

$r = | z_6 |$

$$r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (16\sqrt{2})^2} = \mathbf{16\sqrt{2}}$$

$\beta = \arg(z_6)$

$$\begin{cases} \cos\beta = a/r = 0/8 = 0 \\ \sin\beta = b/r = (16\sqrt{2})/(16\sqrt{2}) = 1 \end{cases}$$

Exercice 9 : 3°)

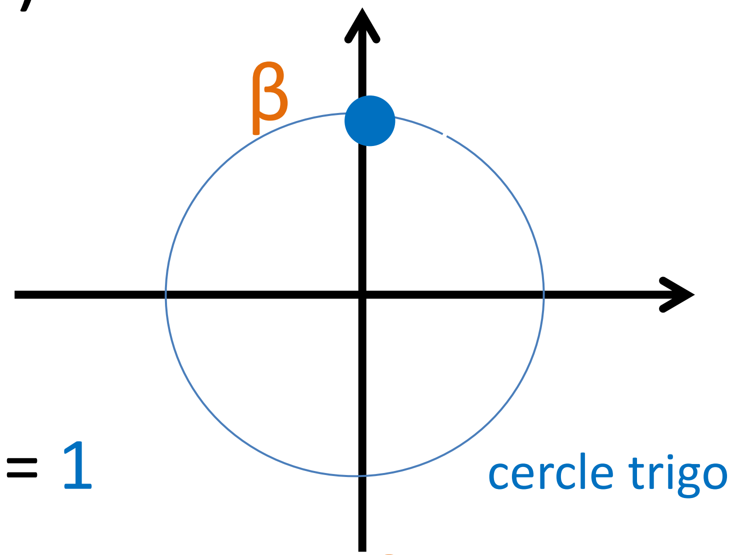
z_6 de **forme algébrique** $z_6 = 16\,i\,\sqrt{2}$

$r = |\, z_6 \,|$

$r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (\,16\sqrt{2}\,)^2} = \mathbf{16\,\sqrt{2}}$

$\beta = \arg(\,z_6\,)$

$$\begin{cases} \cos\beta = a/r = 0/8 = 0 \\ \sin\beta = b/r = (16\sqrt{2})/\,(16\sqrt{2}) = 1 \end{cases}$$



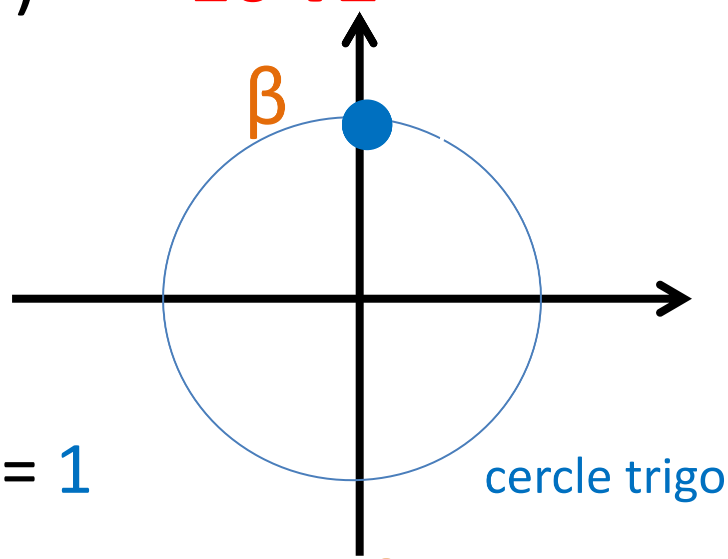
donc $\beta = \boldsymbol{\pi/2} + k2\pi$

Exercice 9 : 3°)

z_6 de **forme algébrique** $z_6 = 16\, i\, \sqrt{2}$

$r = | z_6 |$

$r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (16\sqrt{2})^2} = \mathbf{16\sqrt{2}}$



$\beta = \arg(z_6)$

$$\begin{cases} \cos \beta = a/r = 0/8 = 0 \\ \sin \beta = b/r = (16\sqrt{2})/(16\sqrt{2}) = 1 \end{cases}$$

donc $\beta = \pi/2 + k2\pi$

$Z_6 = 16\, i\, \sqrt{2}$

forme algébrique

$Z_6 = [\, 16\sqrt{2}\, ;\, \pi/2\,]$

forme trigonométrique

3°)

$z_6 = z_4 \times z_5$

$16 i\sqrt{2} = (4 + 4\sqrt{3}i) \times (\sqrt{6} + i\sqrt{2})$

formes algébriques

$[\, 16\sqrt{2} \,;\, \pi/2 \,]$

$= [\, 2\sqrt{2} \,;\, \pi/6 \,] \times [\, 8 \,;\, \pi/3 \,]$

formes trigonométriques

Conjecture : ... ?

3°)

$z_6 = z_4 \, z_5$

$[\, 16\sqrt{2} \,;\, \pi/2 \,]$

$= [\, 2\sqrt{2} \,;\, \pi/6 \,] \times [\, 8 \,;\, \pi/3 \,]$

$= [\, 2\sqrt{2} \times 8 \,;\, \pi/6 + \pi/3 \,]$

Conjecture : $[\, r_1 \,;\, \beta_1 \,] \times [\, r_2 \,;\, \beta_2 \,]$

$= [\, r_1 \times r_2 \,;\, \beta_1 + \beta_2 \,]$?

Elle est vraie et sera démontrée l'année prochaine.

3°)

Conjecture : $[\, r_1 \,;\, \beta_1 \,] \times [\, r_2 \,;\, \beta_2 \,]$

$= [\, r_1 \times r_2 \,;\, \beta_1 + \beta_2 \,]$?

Elle est vraie et sera démontrée l’année prochaine.

On a aussi : …

3°)

Conjecture : $[\, r_1 \,;\, \beta_1 \,] \times [\, r_2 \,;\, \beta_2 \,]$

$= [\, r_1 \times r_2 \,;\, \beta_1 + \beta_2 \,]$?

Elle est vraie et sera démontrée l’année prochaine.

On a aussi : $\dfrac{[\, r_1 \,;\, \beta_1 \,]}{[\, r_2 \,;\, \beta_2 \,]} = \left[\, \dfrac{r_1}{r_2} \,;\, \beta_1 - \beta_2 \,\right]$