Exercice 10 :

Déterminez les formes algébrique et trigonométrique des nombres complexes **z** définis par

1°) $(5 + 6i)^2 = z$

2°) $2z + 25 = i - i(6 + 3z)$

3°) $(z - 1 + i)(z + 3 - 2i) = 20i - 4 + z(1 - i)$

1°) $(5 + 6i)^2 = z$

$z = (5 + 6i)^2$

$= 25 + 60i + (6i)^2$

car $(A + B)^2 = A^2 + 2AB + B^2$

1°) $(5 + 6i)^2 = z$

$$z = (5 + 6i)^2$$

$$= 25 + 60i + (6i)^2$$

$$= 25 + 60i + 36\,i^2$$

$$= 25 + 60i + 36\,(-1)$$

$$= -11 + 60i$$

1°) $(5 + 6i)^2 = z$

z_1 de **forme algébrique** $z_1 = -11 + 60i$

$r = | z_1 |$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-11)^2 + 60^2} = \sqrt{3721} = 61$$

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$\beta = \arg(z_1)$

$$\begin{cases} \cos \beta = a/r = - 11 / 61 \\ \sin \beta = b/r = 60 / 61 \end{cases}$$

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Il n'y a aucun angle remarquable correspondant pour en déduire β en valeur exacte !

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pour en déduire β en valeur exacte ! A la calculatrice :

$$\begin{cases} \beta = \text{Acos}(-11 / 61) \approx 1,75 \text{ rad} \\ \beta = \text{Asin}(60 / 61) \approx 1,39 \text{ rad} \end{cases}$$

$\beta = \ldots ?$

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$$\begin{cases} \cos \beta = a/r = -11 / 61 \approx -0{,}18 \\ \sin \beta = b/r = 60 / 61 \approx 0{,}98 \end{cases}$$

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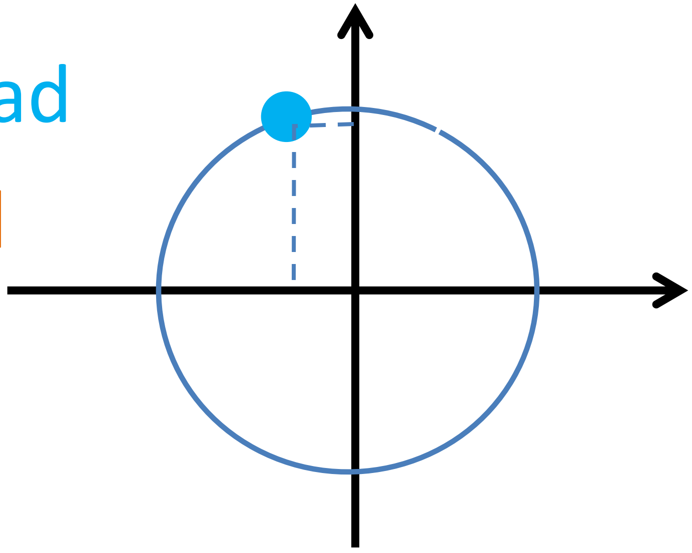

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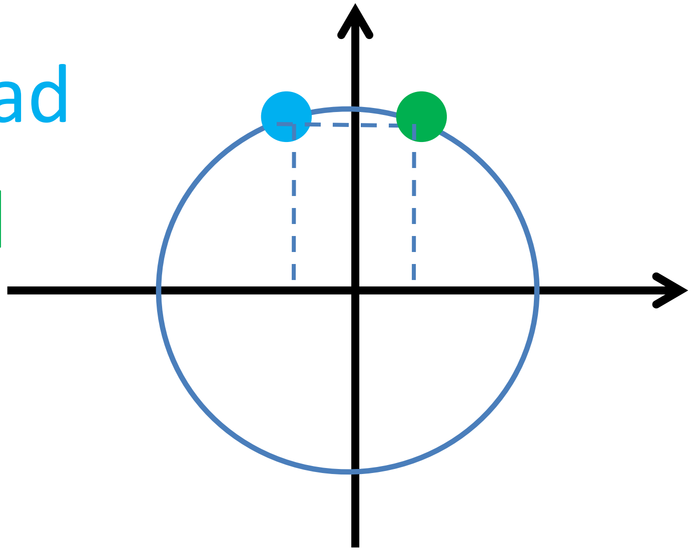

β = … ?

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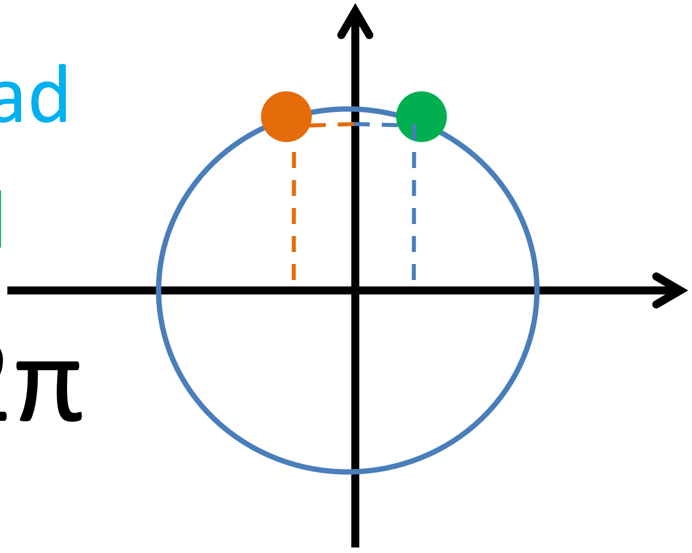

$$\beta \approx 1,75 + k2\pi$$

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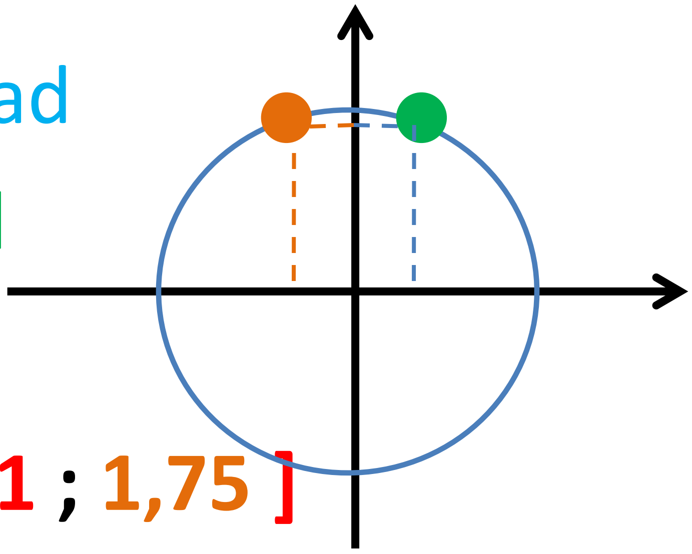

$$\beta \approx 1{,}75 + k2\pi$$

z_1 de **forme trigonométrique** $z_1 \approx [\,61\,;\,1{,}75\,]$

2°) $2z + 25 = i - i\,(6 + 3z)$

$\Leftrightarrow$ …

2°) $2z + 25 = i - i(6 + 3z)$

$\Longleftrightarrow 2z + 25 = i - 6i - 3iz$

$\Longleftrightarrow 2z + 3iz = -25 - 5i$

$\Longleftrightarrow z(2 + 3i) = -25 - 5i$

$\Longleftrightarrow z = \ldots$

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$\Leftrightarrow z\,(2 + 3i) = -25 - 5i$

$$\Leftrightarrow z = \frac{-25 - 5i}{2 + 3i} = \frac{(-25 - 5i)(2 - 3i)}{(2 + 3i)(2 - 3i)}$$

$= \dots$

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$$= \frac{-50 + 75i - 10i + 15i^2}{4 - 9\,i^2}$$

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2°) $2z + 25 = i - i\,(6 + 3z)$

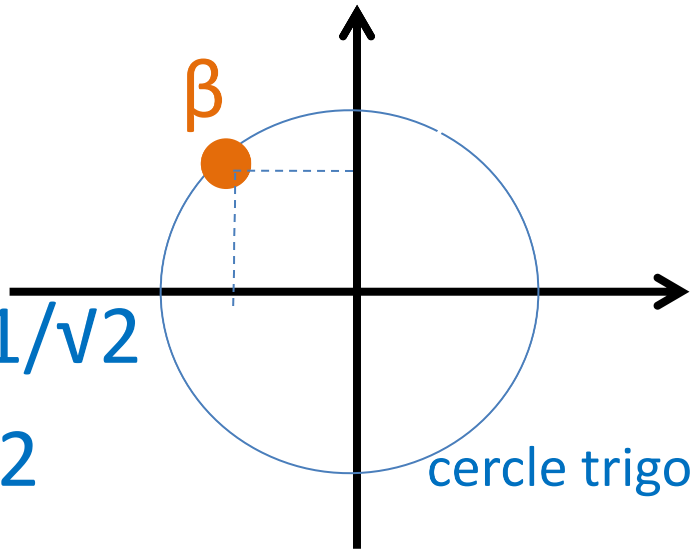
z_2 de **forme algébrique** $z_2 = -5 + 5i$

$r = |\,z_2\,|$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-5)^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

$\beta = \arg(z_2)$

$$\begin{cases} \cos\beta = a/r = -5/(5\sqrt{2}) = -1/\sqrt{2} \\ \sin\beta = b/r = 5/(5\sqrt{2}) = 1/\sqrt{2} \end{cases}$$



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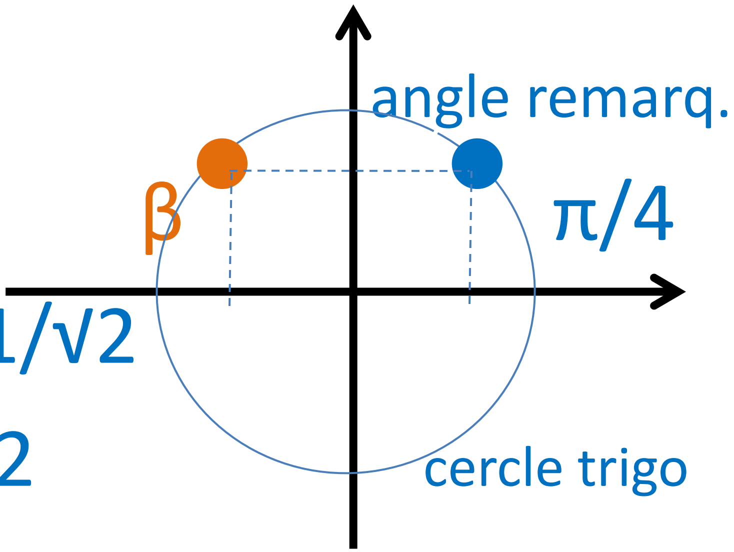

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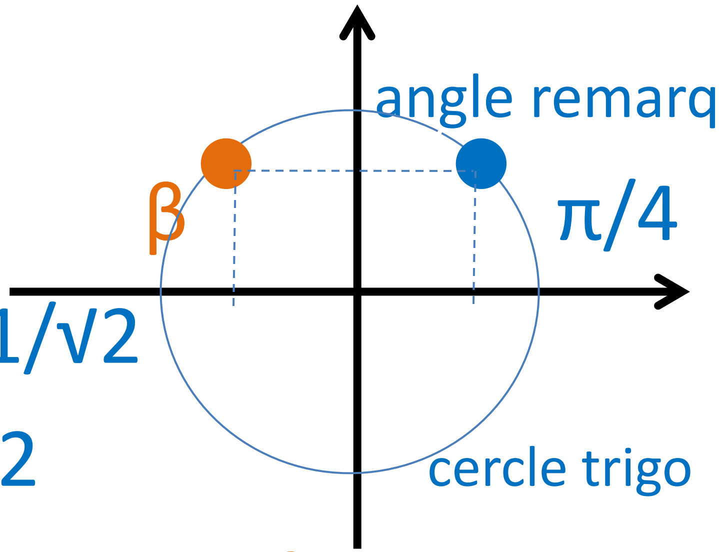

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donc $\beta = 3\pi/4 + k2\pi$

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z_2 de **forme algébrique** $z_2 = -5 + 5i$

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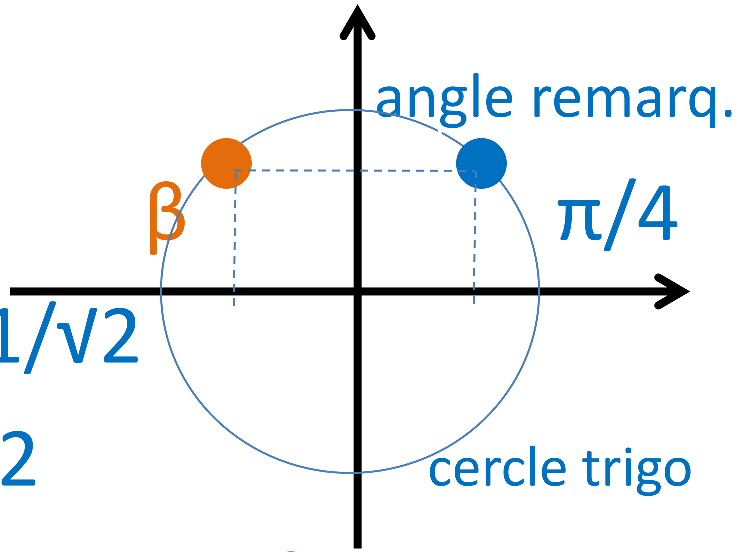

donc $\beta = \mathbf{3\pi/4} + k2\pi$

z_2 de **forme trigonométrique** $z_2 = [\,\mathbf{5\sqrt{2}}\,;\,\mathbf{3\pi/4}\,]$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + (-1 + i)z + (3 - 2i)z$$

$$+ (-1 + i)(3 - 2i)$$

$$- z(1 - i) - 20i + 4 = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i) = 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + (-1 + i + 3 - 2i - 1 + i)z + (-3 + 3i + 2i - 2i^2) - 20i + 4 = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + (1)z$$

$$+ (-1 + 5i)$$

$$- 20i + 4 = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + z + 3 - 15i = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + z + 3 - 15i = 0$$

$$\Leftrightarrow z^2 + 2(0,5)z + 0,5^2 - 0,5^2 + 3 - 15i = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + z + 3 - 15i = 0$$

$$\Leftrightarrow z^2 + 2(0,5)z + 0,5^2 - 0,5^2 + 3 - 15i = 0$$

$$\Leftrightarrow (z + 0,5)^2 - 0,5^2 + 3 - 15i = 0$$

3°)

$$(z - 1 + i)(z + 3 - 2i)$$

$$= 20i - 4 + z(1 - i)$$

$$\Leftrightarrow z^2 + z + 3 - 15i = 0$$

$$\Leftrightarrow z^2 + 2(0,5)z + 0,5^2 - 0,5^2 + 3 - 15i = 0$$

$$\Leftrightarrow (z + 0,5)^2 - 0,5^2 + 3 - 15i = 0$$

$$\Leftrightarrow (z + 0,5)^2 = -2,75 + 15i$$

3°)

$\Leftrightarrow$ $(z + 0,5)^2 = - 2,75 + 15i$

Calculatrice : $\sqrt{- 2,75 + 15i}$

donne $2,5 + 3i$

i se trouve dans OPTN puis CPLX

3°)

$\Leftrightarrow$ $(z + 0{,}5)^2 = -2{,}75 + 15i$

Calculatrice : $\sqrt{-2{,}75 + 15i}$

donne $2{,}5 + 3i$

i se trouve dans OPTN puis CPLX

Vérification :

$(2{,}5 + 3i)^2 = 2{,}5^2 + 2(2{,}5)3i + (3i)^2$

$= 6{,}25 + 15i + 9\,i^2$

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$\Leftrightarrow$ $(z + 0{,}5)^2 = - 2{,}75 + 15i$

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$(2{,}5 + 3i)^2 = 2{,}5^2 + 2(2{,}5)3i + (3i)^2$

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$= 6{,}25 + 15i + 9\ (- 1)$

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Calculatrice : $\sqrt{- 2{,}75 + 15i}$

donne $2{,}5 + 3i$

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Vérification :

$(2{,}5 + 3i)^2 = 2{,}5^2 + 2(2{,}5)3i + (3i)^2$

$= 6{,}25 + 15i + 9 i^2$

$= 6{,}25 + 15i + 9 (- 1) = - 2{,}75 + 15i$

3°)

$\Leftrightarrow$ $(z + 0,5)^2 = - 2,75 + 15i = (2,5 + 3i)^2$

3°)

$\Leftrightarrow$ $(z + 0{,}5)^2 = - 2{,}75 + 15i = (2{,}5 + 3i)^2$

$\Leftrightarrow$ $z + 0{,}5 = 2{,}5 + 3i$

ou $z + 0{,}5 = - (2{,}5 + 3i)$

3°)

$\Leftrightarrow$ $(z + 0,5)^2 = - 2,75 + 15i = (2,5 + 3i)^2$

$\Leftrightarrow$ $z + 0,5 = 2,5 + 3i$

ou $z + 0,5 = - (2,5 + 3i)$

$\Leftrightarrow$ $z = 2 + 3i$

ou $z = - 3 - 3i$

3°)

z_3 de **forme algébrique** $z_3 = 2 + 3i$

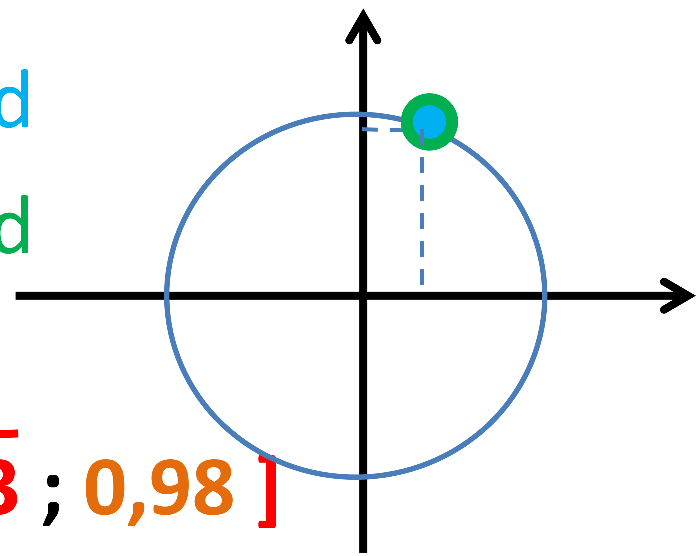
$r = \sqrt{a^2 + b^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$

$$\begin{cases} \cos \beta = a/r = 2 / \sqrt{13} \approx 0{,}55 \\ \sin \beta = b/r = 3 / \sqrt{13} \approx 0{,}83 \end{cases}$$

Il n'y a aucun angle remarquable correspondant pour en déduire β en valeur exacte ! A la calculatrice :

$$\begin{cases} \beta = \text{Acos}\,(2 / \sqrt{13}) \approx 0{,}98 \text{ rad} \\ \beta = \text{Asin}\,(3 / \sqrt{13}\,) \approx 0{,}98 \text{ rad} \end{cases}$$

$$\beta \approx 0{,}98 + k2\pi$$



z_3 de **forme trigonométrique** $\approx [\ \sqrt{13}\ ;\ 0{,}98\ ]$

3°)

z_4 de **forme algébrique** $z_4 = -3 - 3i$

$r = | z_4 |$

$r = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = \mathbf{3\sqrt{2}}$

$\beta = \arg(z_4)$

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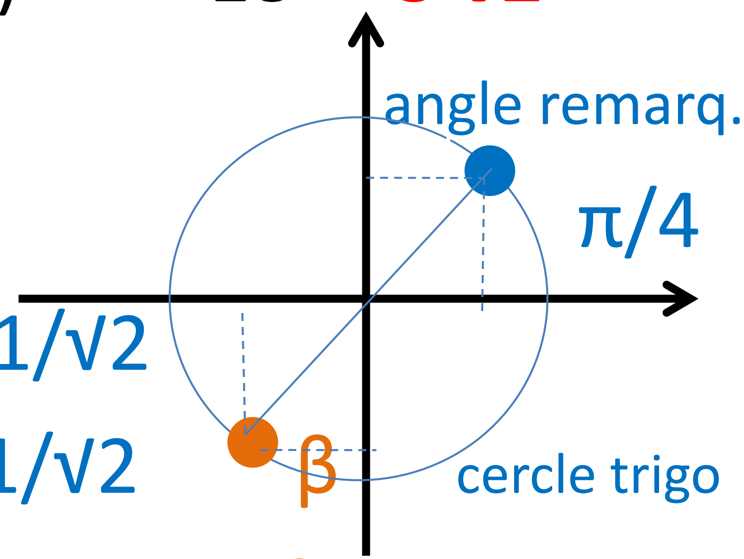

donc $\beta = \mathbf{5\pi/4} + k2\pi$

z_4 de **forme trigonométrique** $z_4 = [\ \mathbf{3\sqrt{2}}\ ;\ \mathbf{5\pi/4}\ ]$