Exercice 11 :

Soient dans le plan

muni du repère orthonormé $(O ; \vec{i} ; \vec{j})$

les points A, B et C d'affixes $z_A = 1 + i$

$z_B = (1 + \sqrt{3}) i$ et $z_C = 2\sqrt{3} + 1 + 3i$

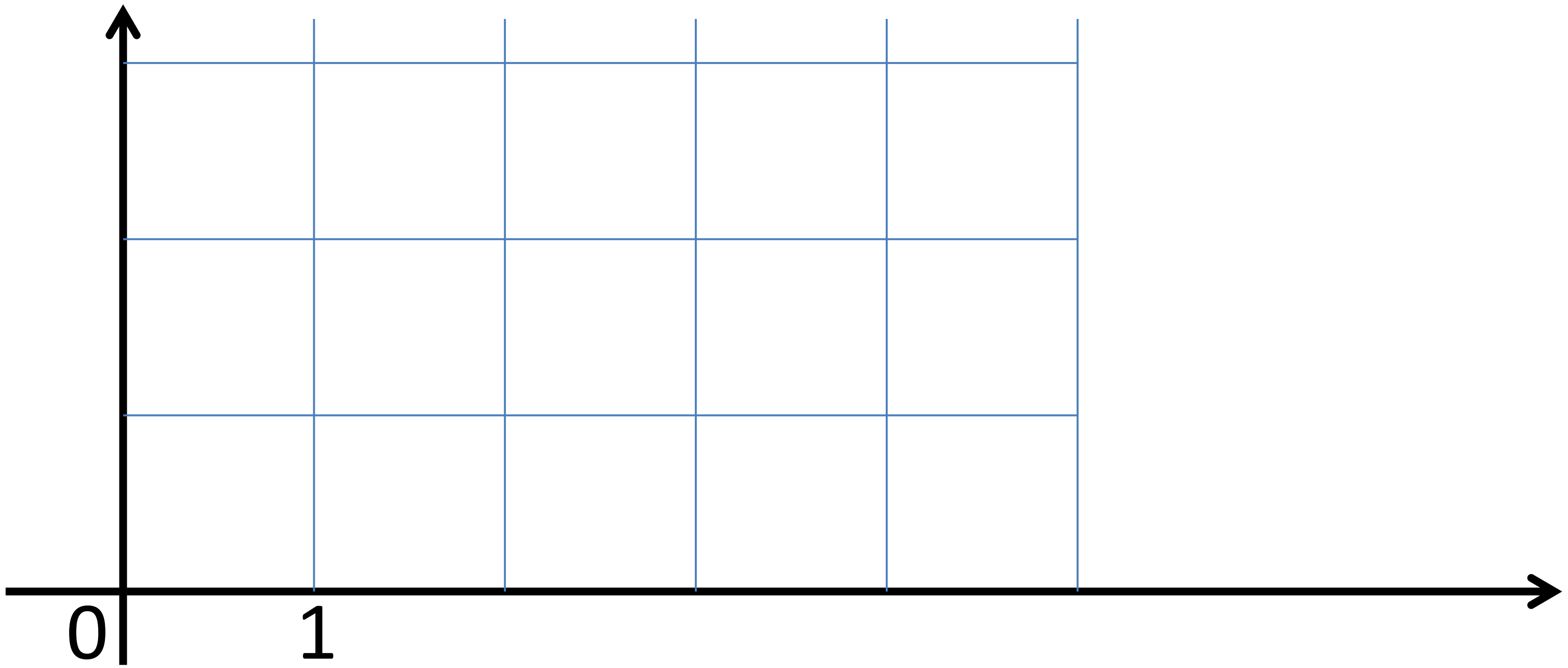
1°) Déterminez les angles orientés

$(\vec{i} ; \overrightarrow{AB})$ et $(\vec{i} ; \overrightarrow{AC})$.

2°) Déduisez-en l'angle orienté $(\overrightarrow{AB} ; \overrightarrow{AC})$

et la nature du triangle ABC.

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$

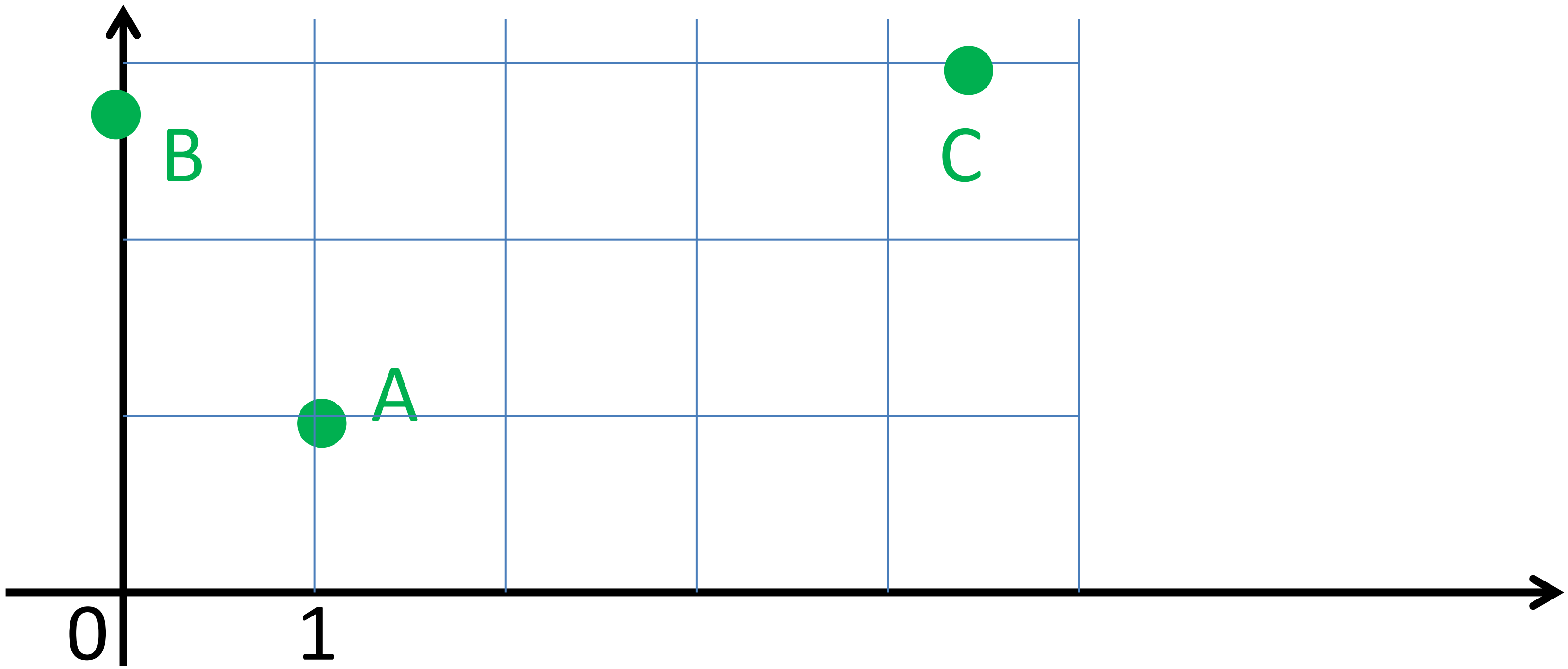


$z_A = 1 + i$ est l'affixe du point A(;)

$z_B = (1 + \sqrt{3})\, i$ est l'affixe du point B(;)

$z_C = 2\sqrt{3} + 1 + 3i$ est l'affixe du point C(;)

1°) $z_A = \mathbf{1 + i}$; $z_B = \mathbf{(1 + \sqrt{3})\, i}$; $z_C = \mathbf{2\sqrt{3} + 1 + 3i}$

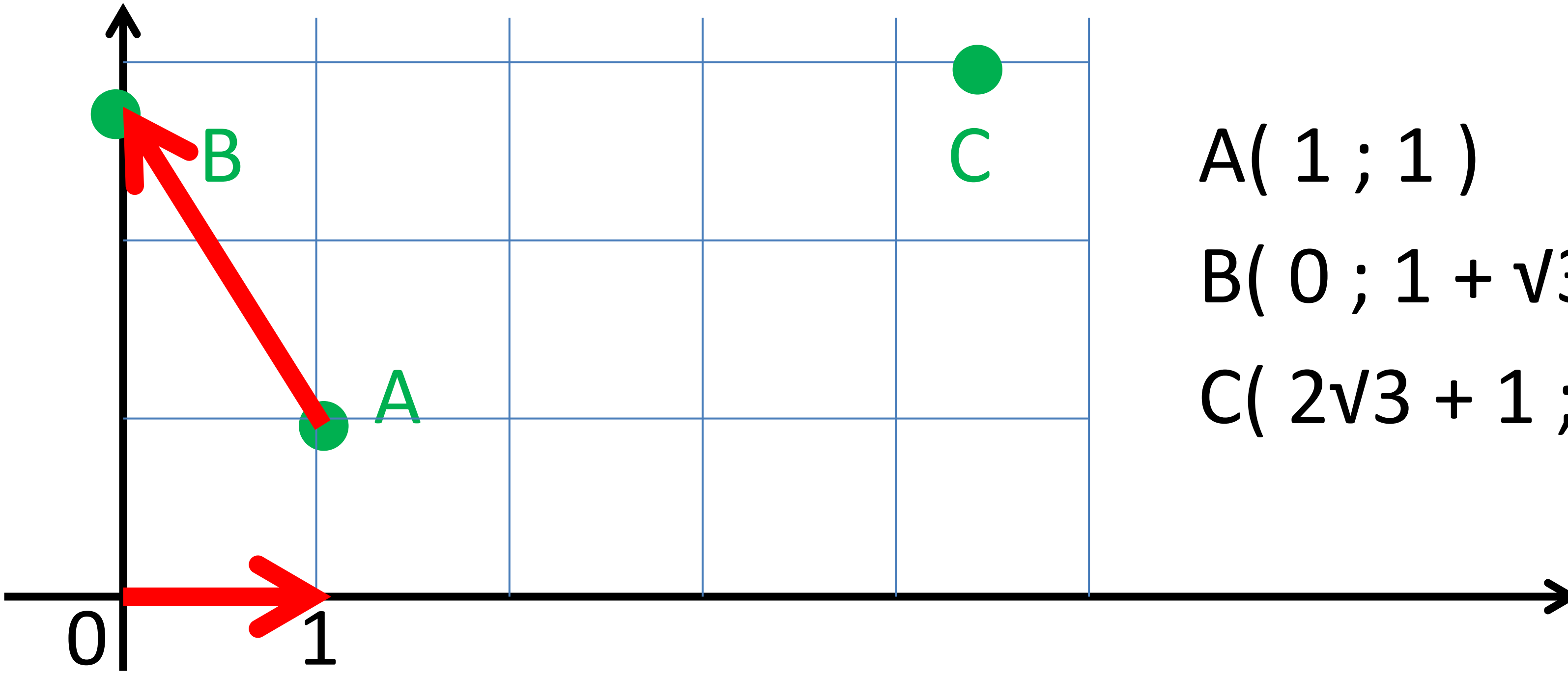


$z_A = \mathbf{1 + i}$ est l'affixe du point A(1 ; 1)

$z_B = \mathbf{(1 + \sqrt{3})\, i}$ est l'affixe du point B(0 ; 1 + √3)

$z_C = \mathbf{2\sqrt{3} + 1 + 3i}$ est l'affixe du point C(2√3 + 1 ; 3)

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})i$; $z_C = 2\sqrt{3} + 1 + 3i$



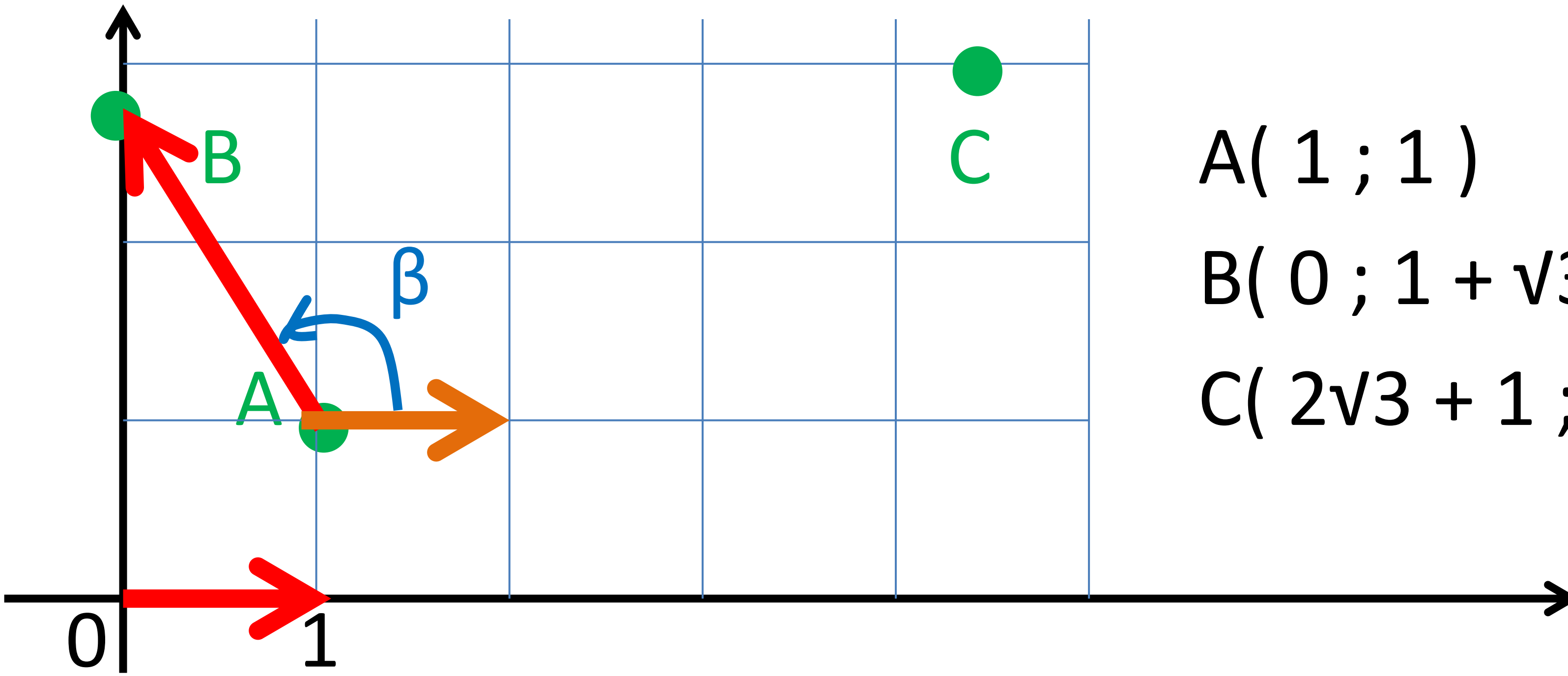
A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

$(\vec{i} \,;\, \overrightarrow{AB}) = \beta$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



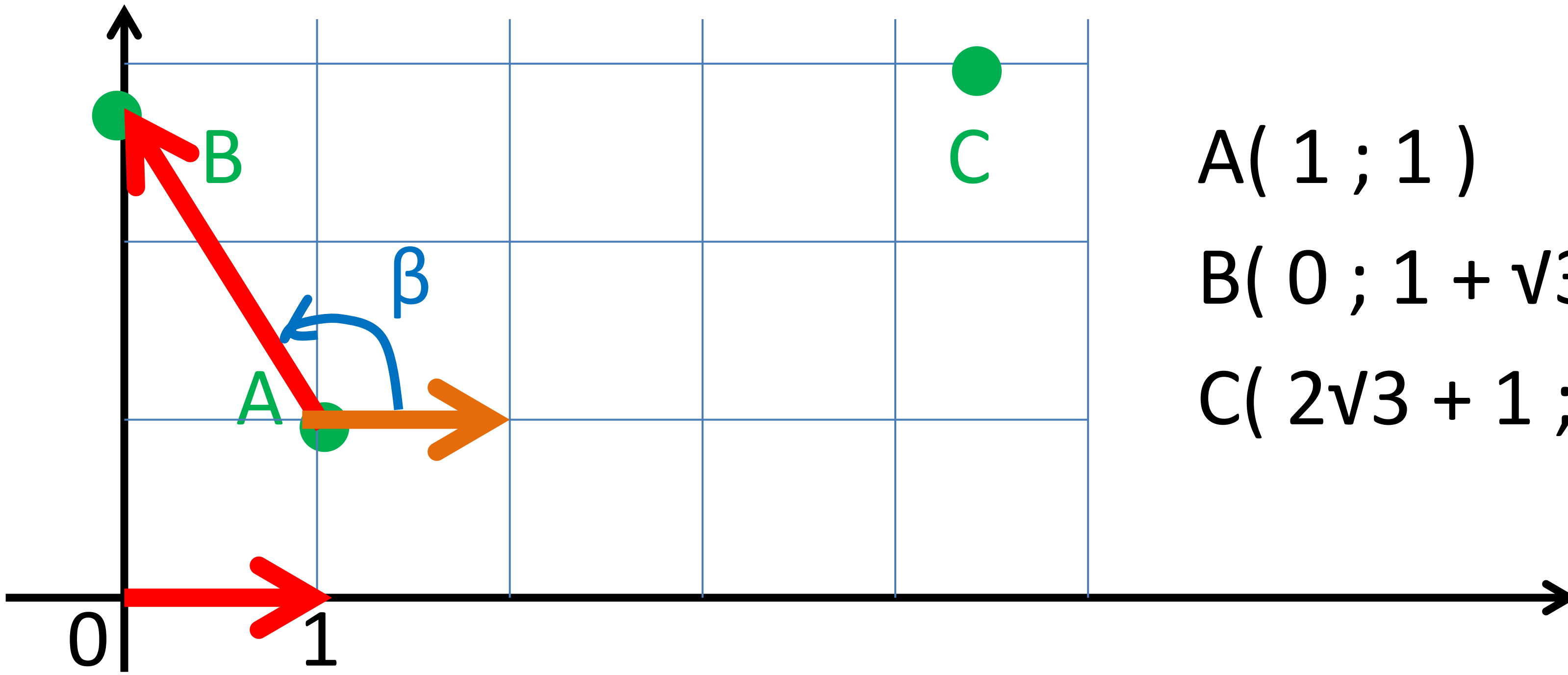
A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

$(\vec{i} \,;\, \overrightarrow{AB}) = \beta$ donc $\beta = \arg(z_{\overrightarrow{AB}})$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

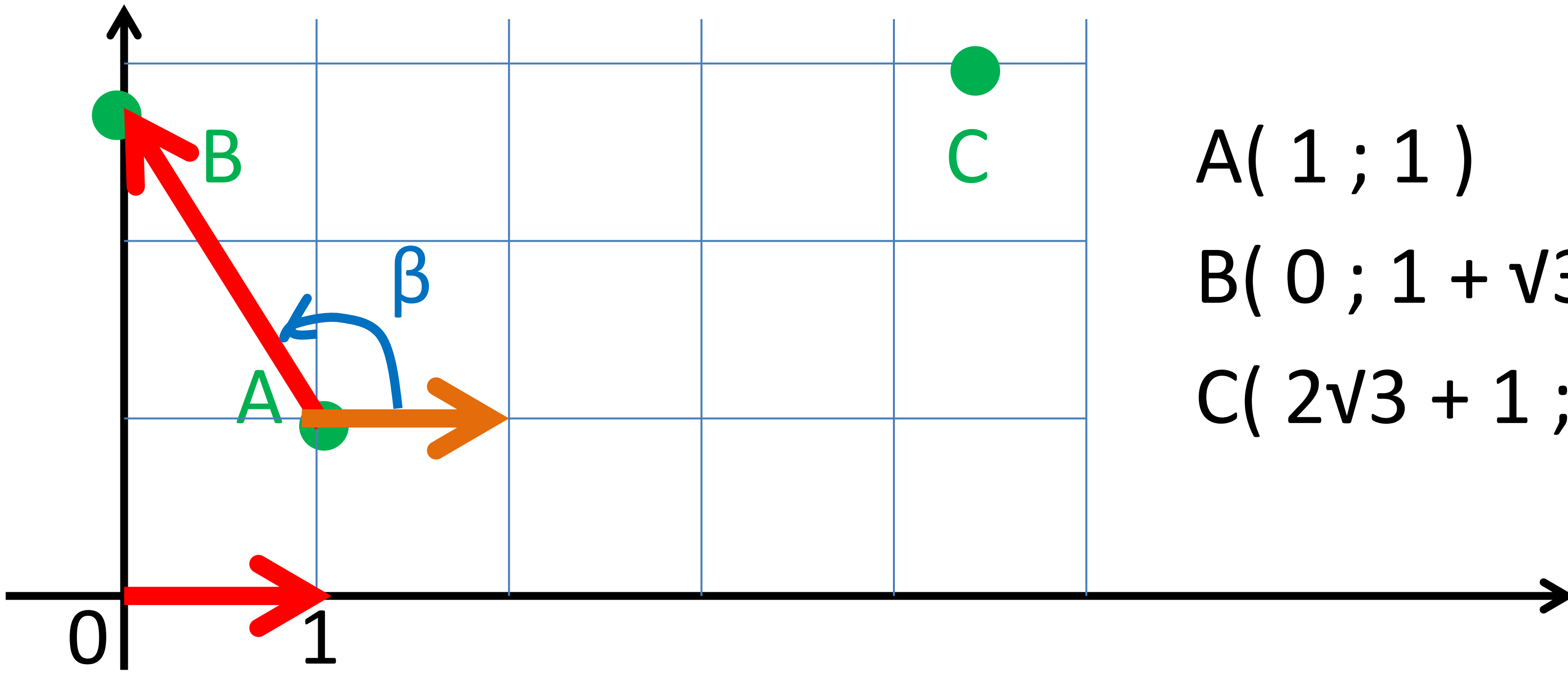
$(\vec{i} ; \overrightarrow{AB}) = \beta$ $\overrightarrow{AB}(0 - 1 ; 1 + \sqrt{3} - 1) = (-1 ; \sqrt{3})$

$\overrightarrow{AB}$ a pour affixe $z_{\overrightarrow{AB}} = -1 + i\sqrt{3}$

r = ...

cos β = ... et sin β = ...

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

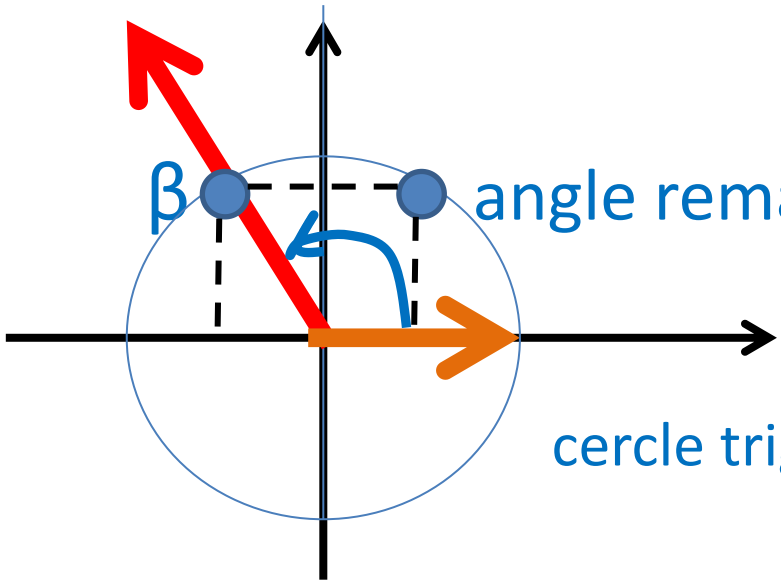
$(\vec{i}\,;\,\overrightarrow{AB}) = \beta$ $\quad \overrightarrow{AB}(0 - 1 \,;\, 1 + \sqrt{3} - 1) = (-1 \,;\, \sqrt{3})$

$\overrightarrow{AB}$ a pour affixe $z_{\overrightarrow{AB}} = -1 + i\sqrt{3}$

$r = |\, z_{\overrightarrow{AB}} \,| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$

$\cos\beta = a/r = -\frac{1}{2}$ et $\sin\beta = b/r = (\sqrt{3})/2$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



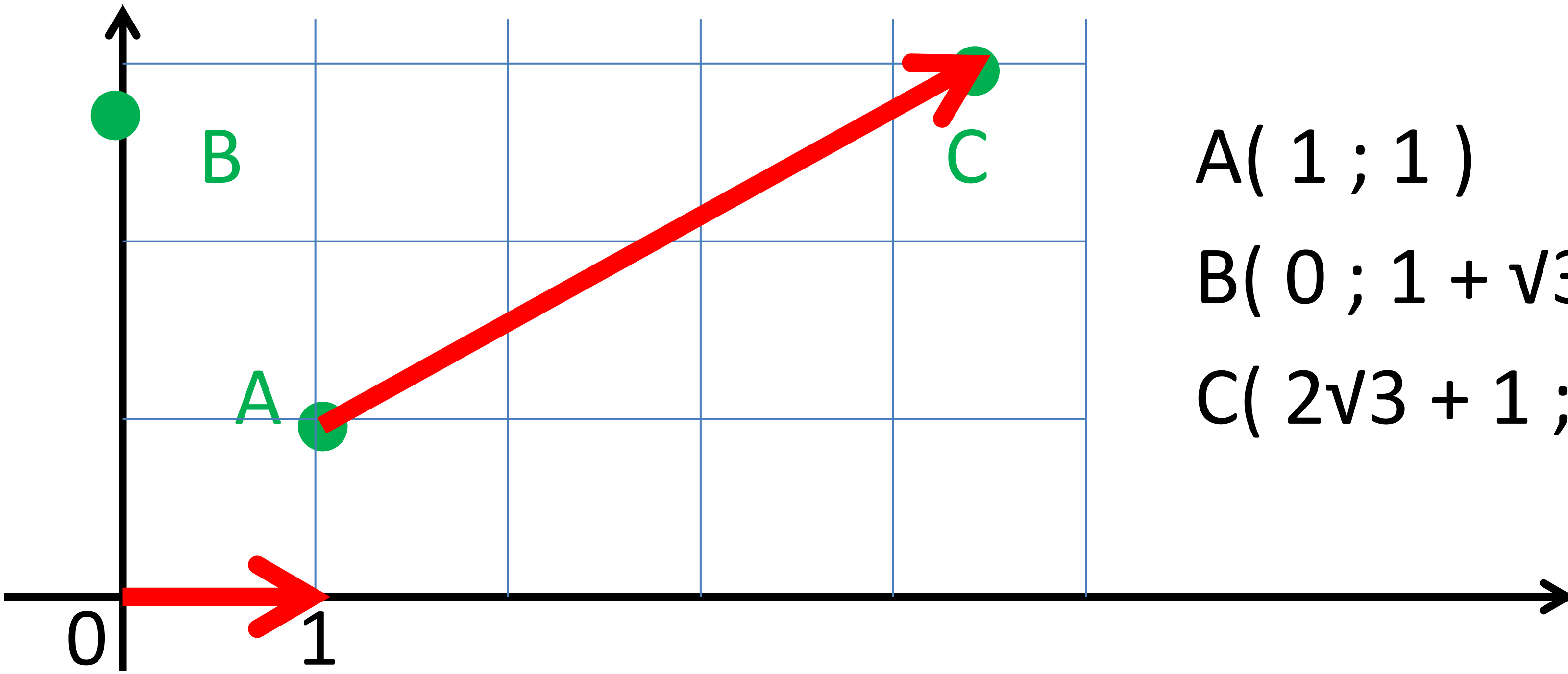
$(\vec{i} ; \overrightarrow{AB}) = \beta$ $\overrightarrow{AB}(0 - 1 ; 1 + \sqrt{3} - 1) = (-1 ; \sqrt{3})$

$\overrightarrow{AB}$ a pour affixe $z_{\overrightarrow{AB}} = -1 + i\sqrt{3}$

$r = |z_{\overrightarrow{AB}}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$

$\cos\beta = -\frac{1}{2}$ et $\sin\beta = (\sqrt{3})/2$ donc $\beta = 2\pi/3 + k2\pi$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



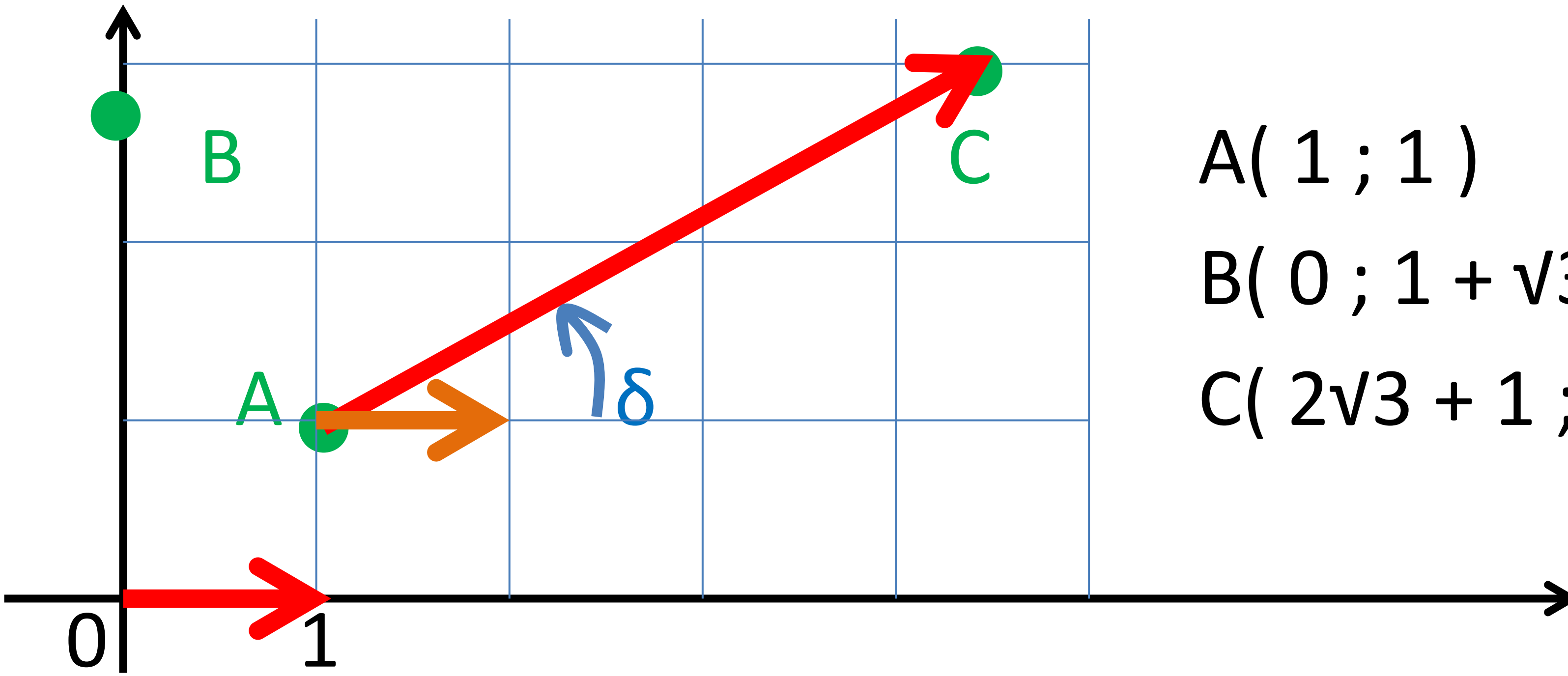
A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

$(\vec{i} \,;\, \vec{AC}) = \delta$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



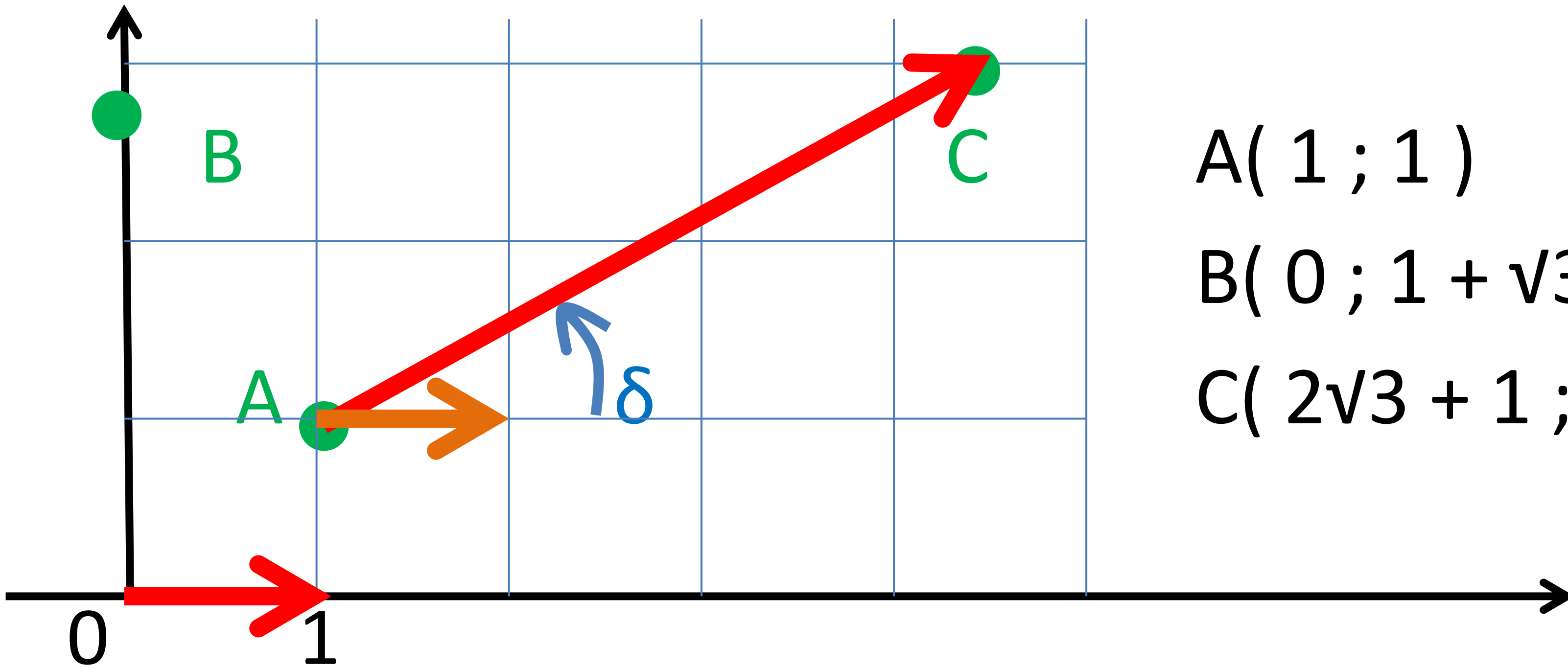
A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

$(\vec{i}\,;\overrightarrow{AC}) = \delta$ donc $\delta = \arg(z_{\overrightarrow{AC}})$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



A(1 ; 1)

B(0 ; 1 + √3)

C(2√3 + 1 ; 3)

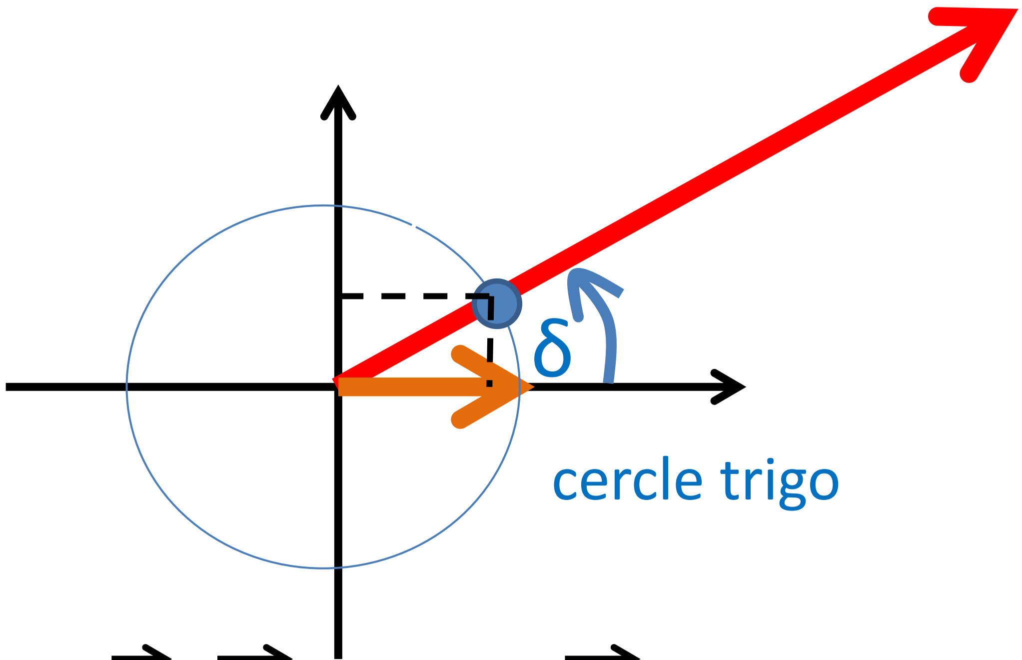
$(\vec{i}\,;\,\overrightarrow{AC}) = \delta$ $\quad \overrightarrow{AC}(\,2\sqrt{3} + 1 - 1\,;\,3 - 1\,) = (\,2\sqrt{3}\,;\,2\,)$

AC a pour affixe $z_{\overrightarrow{AC}} = 2\sqrt{3} + 2i$

$r = |\,z_{\overrightarrow{AC}}\,| = \sqrt{(2\sqrt{3})^2 + (2)^2} = \sqrt{12 + 4} = 4$

$\cos\delta = 2(\sqrt{3})/4 = (\sqrt{3})/2$ et $\sin\delta = 2/4 = ½$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



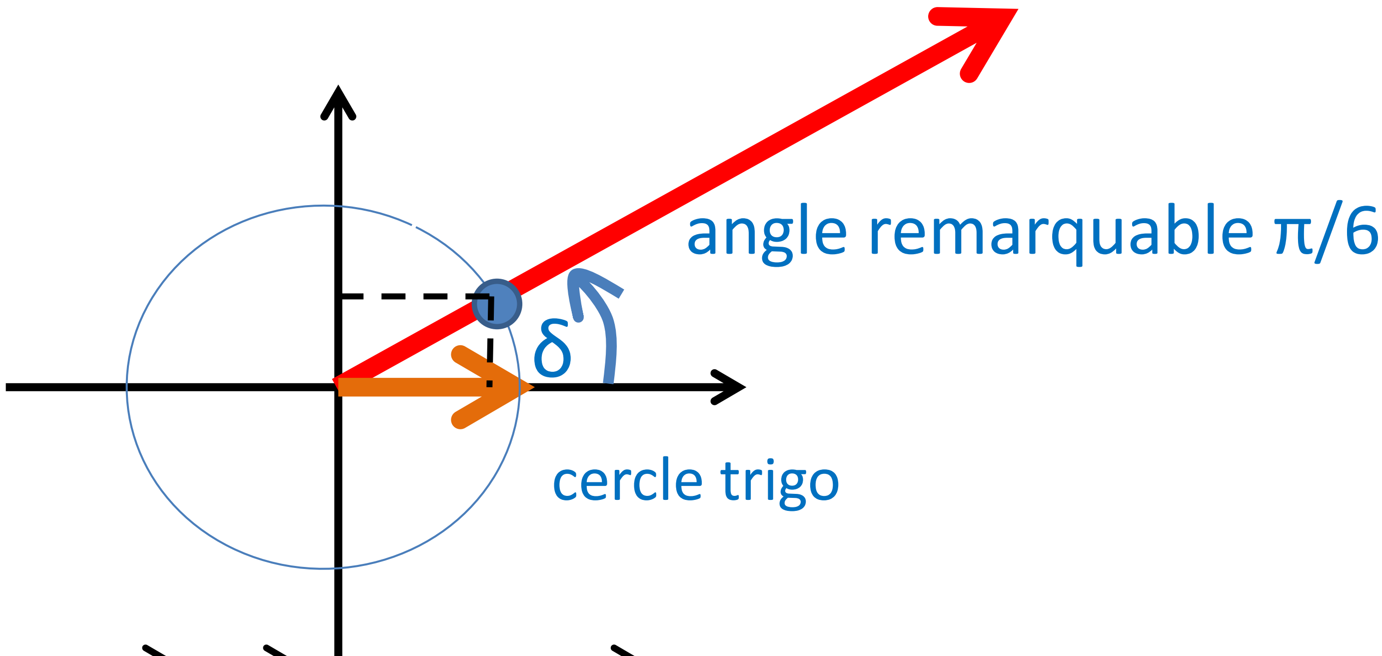
$(\vec{i}\,;\overrightarrow{AC}) = \delta$ $\overrightarrow{AC}(2\sqrt{3} + 1 - 1\,;\,3 - 1) = (2\sqrt{3}\,;\,2)$

$\overrightarrow{AC}$ a pour affixe $z_{\overrightarrow{AC}} = 2\sqrt{3} + 2i$

$r = |z_{\overrightarrow{AC}}| = \sqrt{(2\sqrt{3})^2 + (2)^2} = \sqrt{12 + 4} = 4$

$\cos\delta = (\sqrt{3})/2$ et $\sin\delta = ½$ donc $\delta = \ldots$

1°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



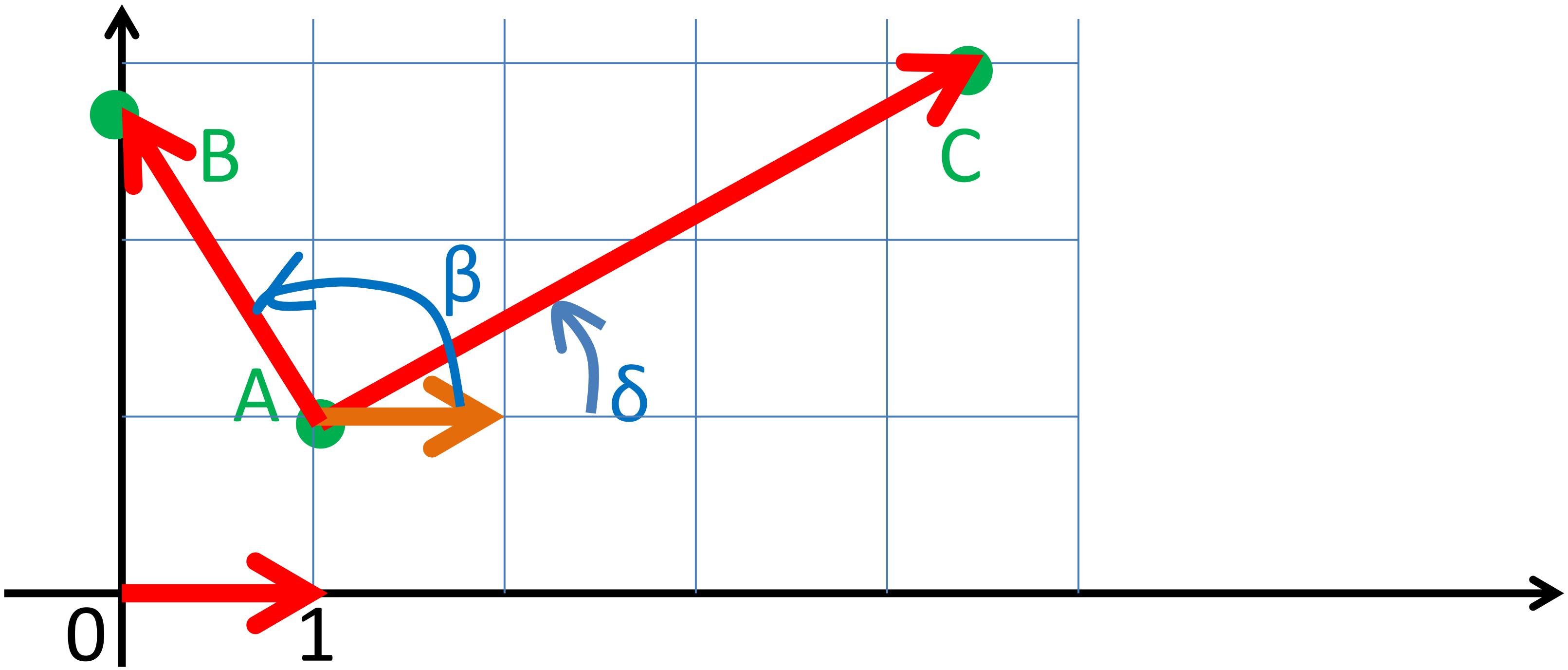
$(\vec{i} ; \vec{AC}) = \delta$ $\quad \vec{AC}(2\sqrt{3} + 1 - 1 ; 3 - 1) = (2\sqrt{3} ; 2)$

$\vec{AC}$ a pour affixe $z_{\vec{AC}} = 2\sqrt{3} + 2i$

$r = | z_{\vec{AC}} | = \sqrt{(2\sqrt{3})^2 + (2)^2} = \sqrt{12 + 4} = 4$

$\cos \delta = (\sqrt{3})/2$ et $\sin \delta = ½$ donc $\delta = \pi/6 + k2\pi$

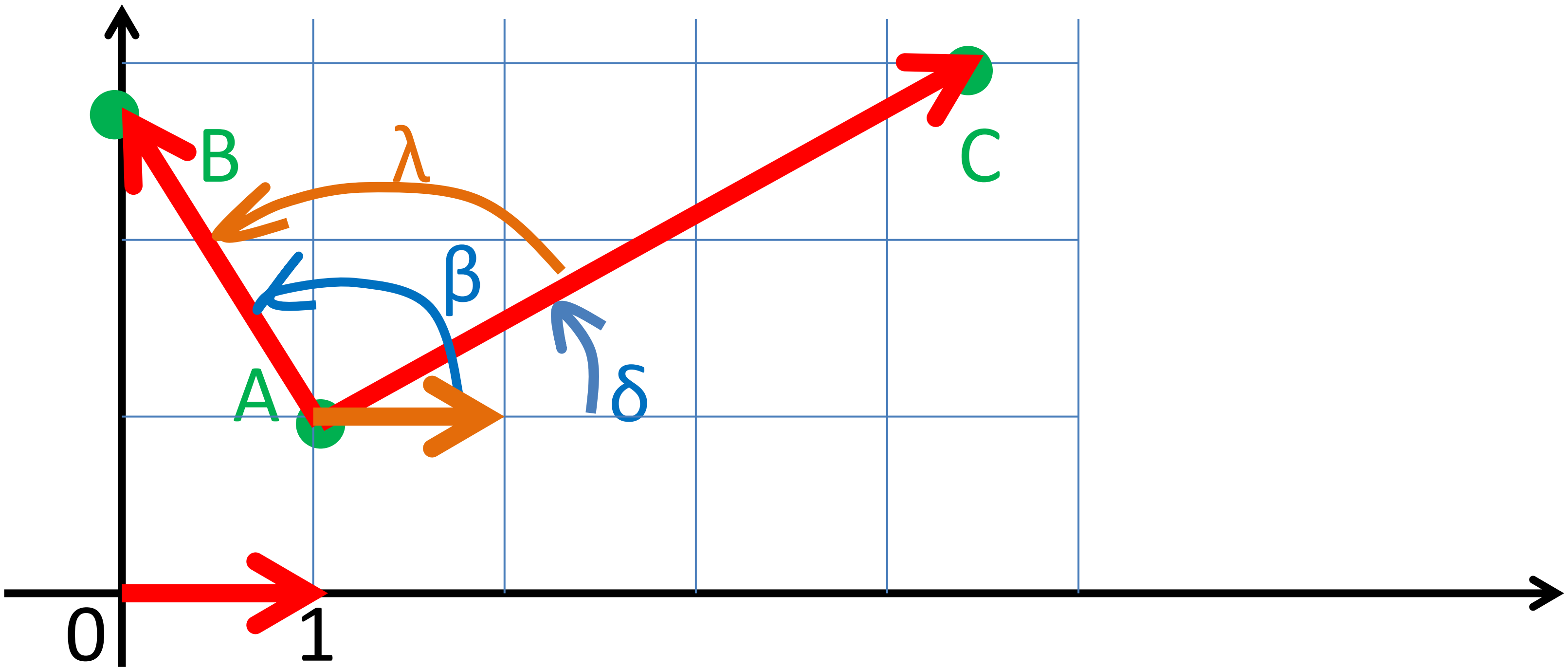
2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



$(\vec{i}\,;\,\overrightarrow{AB}) = \beta = 2\pi/3$ $(\vec{i}\,;\,\overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC}\,;\,\overrightarrow{AB}) = \ldots$

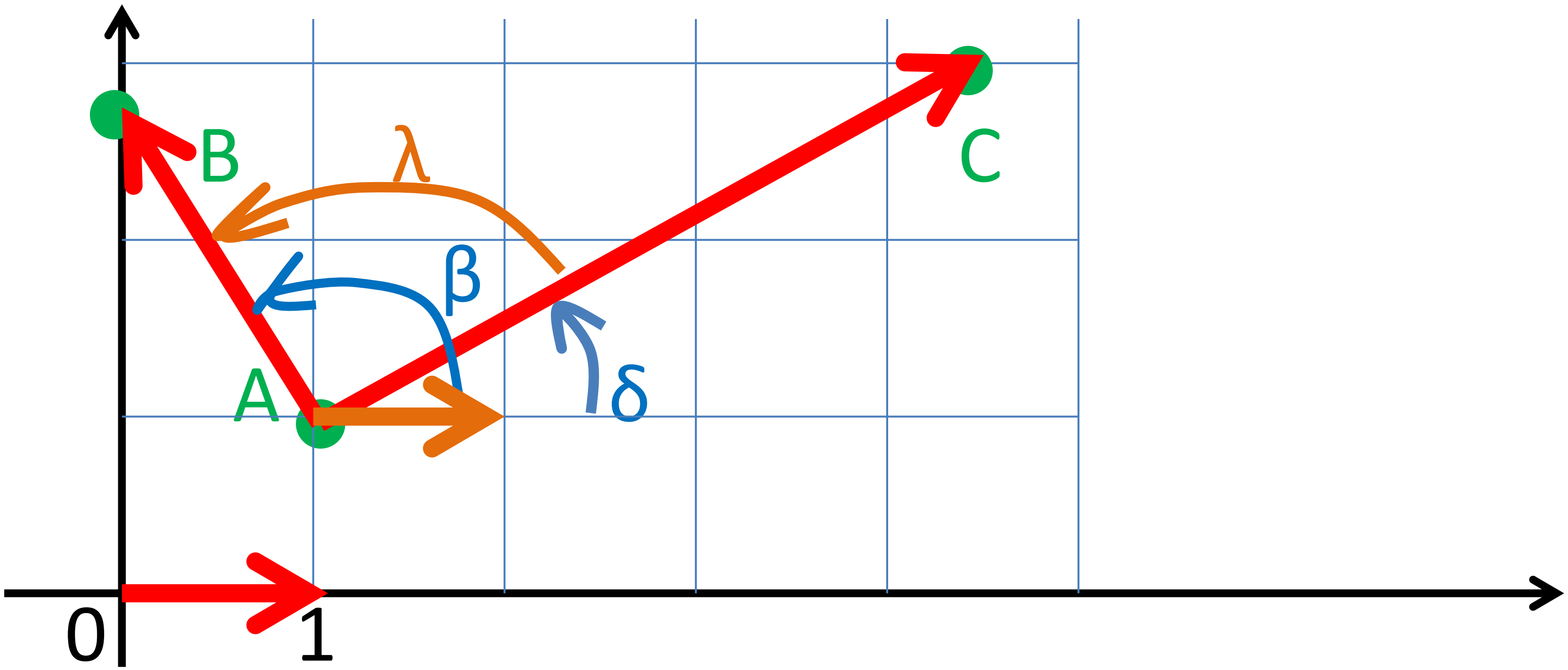
2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



$(\vec{i}\,;\,\overrightarrow{AB}) = \beta = 2\pi/3$ $(\vec{i}\,;\,\overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC}\,;\,\overrightarrow{AB}) = \lambda = \ldots$

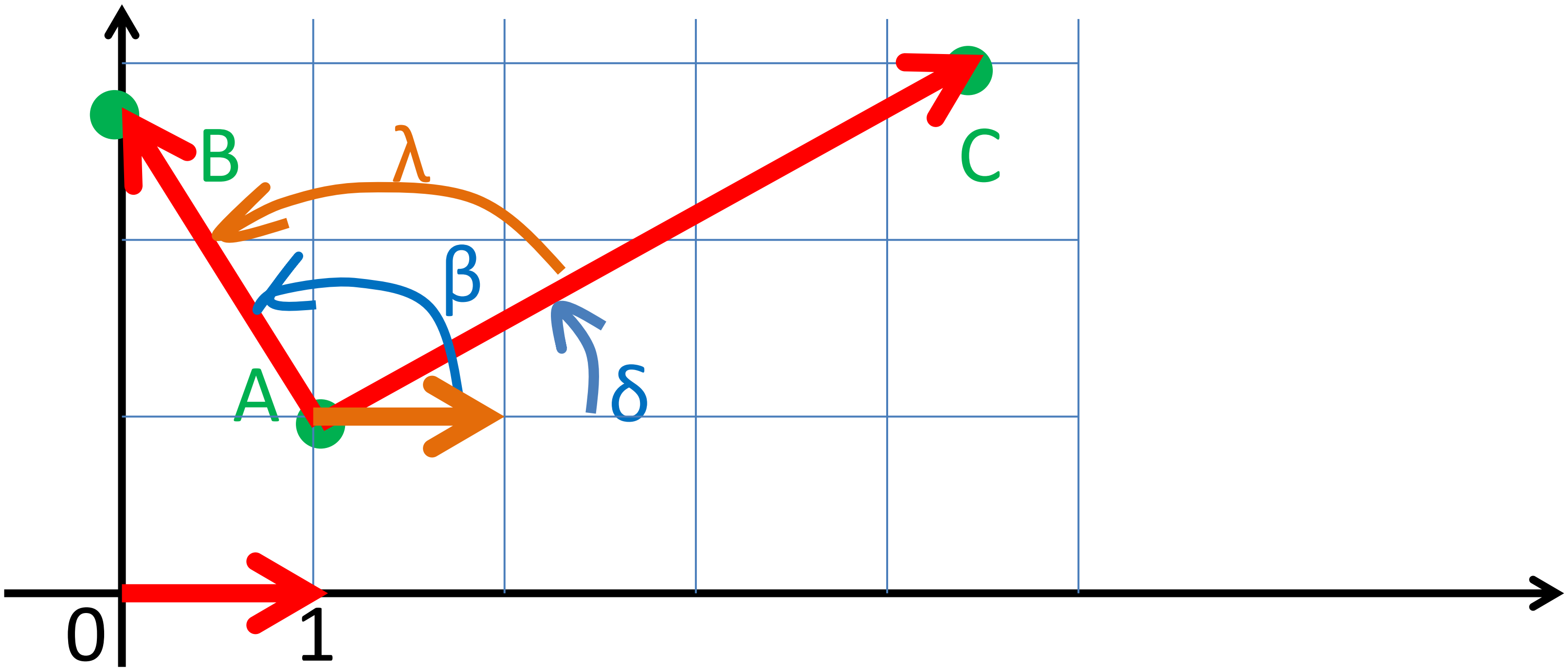
2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



$(\vec{i}\,;\overrightarrow{AB}) = \beta = 2\pi/3$ $\qquad$ $(\vec{i}\,;\overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC}\,;\overrightarrow{AB}) = \lambda = \beta - \delta = \ldots$

2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$

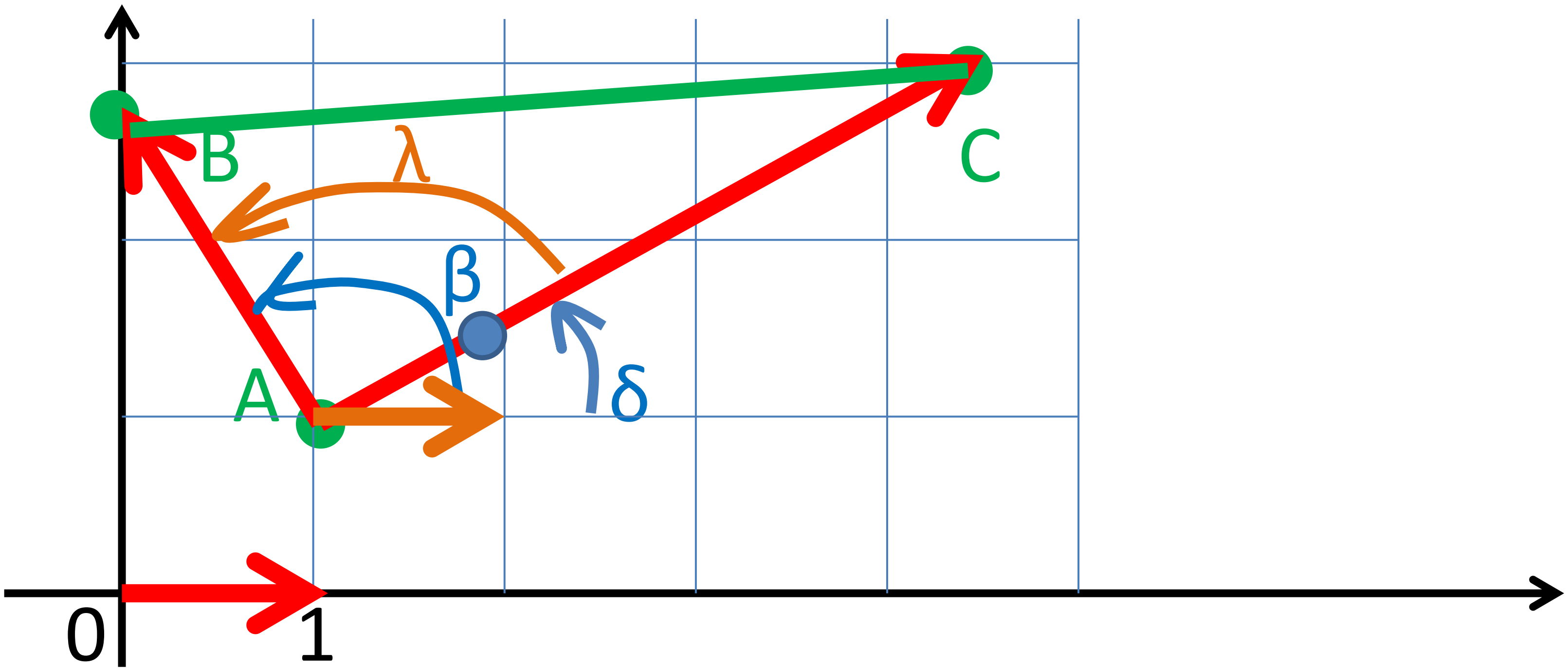


$(\vec{i}\,;\,\overrightarrow{AB}) = \beta = 2\pi/3$ $(\vec{i}\,;\,\overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC}\,;\,\overrightarrow{AB}) = \lambda = \beta - \delta = 2\pi/3 - \pi/6$

$= 4\pi/6 - \pi/6 = 3\pi/6 = \pi/2$

2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\,i$; $z_C = 2\sqrt{3} + 1 + 3i$



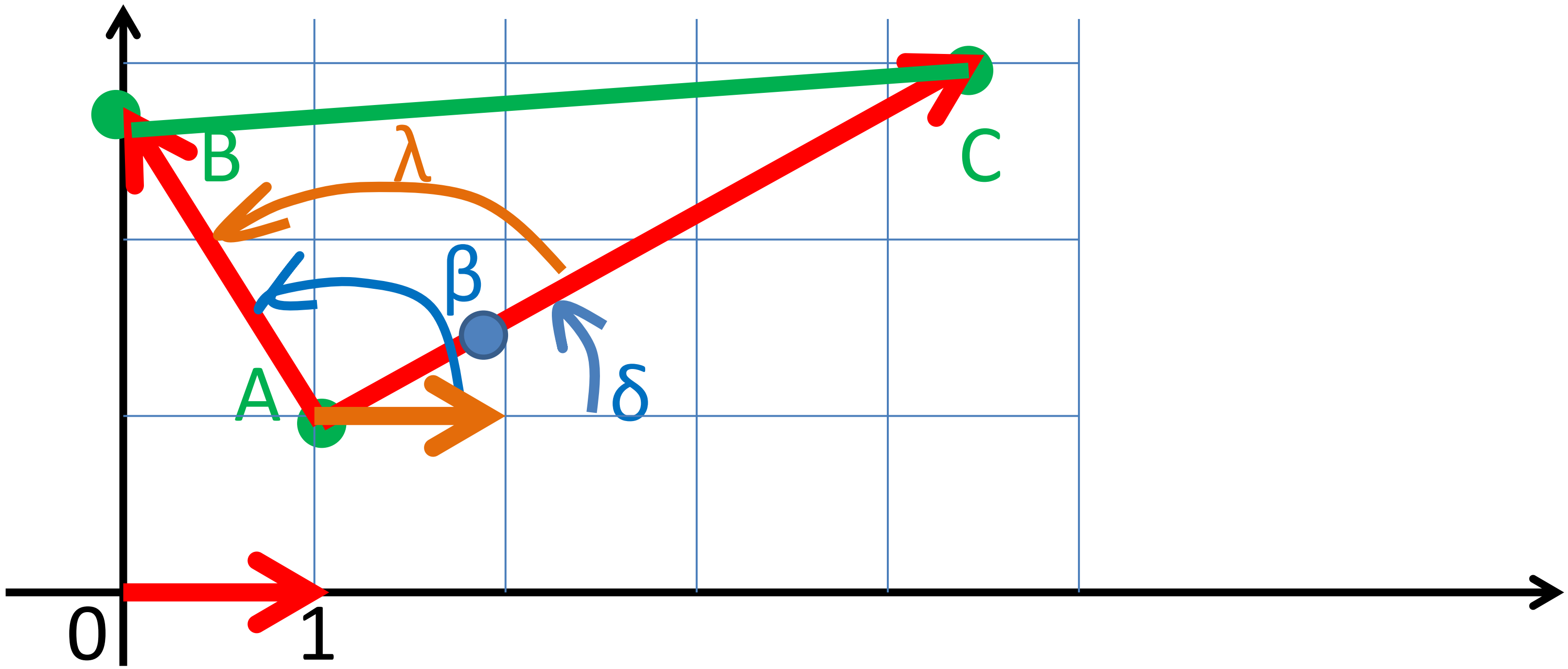
$(\vec{i}\,;\overrightarrow{AB}) = \beta = 2\pi/3$ $\qquad$ $(\vec{i}\,;\overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC}\,;\overrightarrow{AB}) = \lambda = \beta - \delta = 2\pi/3 - \pi/6$

$= 4\pi/6 - \pi/6 = 3\pi/6 = \pi/2$ radian

donc …

2°) $z_A = 1 + i$; $z_B = (1 + \sqrt{3})\, i$; $z_C = 2\sqrt{3} + 1 + 3i$



$(\vec{i} ; \overrightarrow{AB}) = \beta = 2\pi/3$ $(\vec{i} ; \overrightarrow{AC}) = \delta = \pi/6$

donc $(\overrightarrow{AC} ; \overrightarrow{AB}) = \lambda = \beta - \delta = 2\pi/3 - \pi/6$

$= 4\pi/6 - \pi/6 = 3\pi/6 = \pi/2$ radian

donc le triangle ABC est rectangle en A.